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Series and Summations on Binomial Coefficients of Optimized Combination

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Abstract
This paper presents innovative series and summations derived from the optimized combinations relating to the combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems.
Keywords: optimized combination, binomial coefficient, binomial series.

1. Introduction to Optimized Combination

The optimized combination (Annamalai et al., 2020) applied on the computation of multiple geometric series (Annamalai, 2010) is expressed as follows:

$$V_r^n = \frac{(r+1)(r+2)\cdots\cdots(r+n)}{n!} = \frac{(n+1)(n+2))\cdots\cdots(n+r)}{r!} = V_n^r,$$

$$i.e., \qquad V_r^n = \prod_{i=1}^{n} \frac{r+i}{n!} = \prod_{i=1}^{r} \frac{n+i}{r!} = V_n^r \quad (n, r \geq 1\ \&\ n, r \in N),$$

where $N = \{0, 1, 2, 3, \cdots\cdots\}, V_r^n$ is a binomial coefficient, and $n!$ is the factorial of n.

Some results (Annamalai et al., 2020) of the optimized combination are provided below:

i). $V_n^0 = V_0^n = 1\ (n \geq 1\ \ \&\ \ n \in N),$

where V_n^0 alaway implies V_0^n , $i.e., V_n^0 \Rightarrow V_0^n$.

Note that $V_r^n = V_n^r = (n+r)C_r = (n+r)C_n = \dfrac{(n+r)!}{n!\,r!}$ and $V_0^0 = 1$.

ii). $V_r^n = V_n^r$ $(n, r \geq 1$ & $n, r \in N)$ & $V_n^0 = V_0^n$.

iii): $V_0^n + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \Rightarrow \sum_{i=0}^{n} V_i^n = V_r^{n+1}$ $(n, r \in N)$.

2. Novel Series of Optimized Combination

From the result (iii) in this paper, that is (Annamalai, 2020),

$$1 + V_1^n + V_2^n + V_3^n + \cdots + V_r^n = V_r^{n+1} \Leftrightarrow 1 + V_n^1 + V_n^2 + V_n^3 + \cdots + V_n^r = V_{n+1}^r,$$

the following series and its summations (Annamalai, 2018 & Annamalai, 2019) are expressed:

(1). $$\sum_{i=0}^{n} \frac{(i+1)}{1!} = 1 + 2 + 3 + \cdots + n + (n+1) = \frac{(n+1)(n+2)}{2!}.$$

(2). $$\sum_{i=0}^{n} \frac{(i+1)(i+2)}{2!} = 1 + 3 + 6 + \cdots + \frac{(n+1)(n+2)}{2!} = \frac{(n+1)(n+2)(n+3)}{3!}.$$

(3). $$\sum_{i=0}^{n} \frac{(i+1)(i+2)(i+3)}{3!} = \frac{(n+1)(n+2)(n+3)(n+4)}{4!}.$$

(4). $$\sum_{i=0}^{n} \frac{(i+1)(i+2)(i+3)(i+4)}{4!} = \frac{(n+1)(n+2)(n+3)(n+4)(n+5)}{5!}.$$

Similarly, the series continues upto *r* times. The *r*th series and its summation are:

(r). $$\sum_{i=0}^{n} \frac{(i+1)(i+2)(i+3)\cdots(i+r)}{r!} = \frac{(n+1)(n+2)\cdots(n+r)(n+r+1)}{(r+1)!}$$

$$i.e., \sum_{i=0}^{n} \prod_{j=1}^{r} \frac{i+j}{r!} = \prod_{i=1}^{r+1} \frac{n+i}{(r+1)!}.$$

3. Conclusion

In this paper, the innovative series and summations of binomial coefficients have been derived using the results (Annamalai, 2022) of the optimized combination in the field of combinatorics. These series and summations will be useful for the researchers who are involving to solve the scientific problems and meet today's challenges (Annamalai, 2010).

References

Annamalai, C., (2018), *A Model of Iterative Computations, for Recursive Summability*, Tamsui Oxford Journal of Information and Mathematical Sciences, Taiwan, 32(1), 75-77 & Mathematical Reviews, American Mathematical Society. United States of America, MathSciNet: https://mathscinet.ams.org/mathscinet-getitem?mr=3982555.

Annamalai, C., (2018), *Annamalai's Computing Model for Algorithmic Geometric Series and its Mathematical Structures*, Journal of Mathematics and Computer Science, USA, 3(1), 1-6.

Annamalai, C., Srivastava, H. M., Mishra, V. N., (2019), *Recursive Computations and Differential and Integral Equations for Summability of Binomial Coefficients with Combinatorial Expressions*, International Journal of Scientific Research in Mechanical and Materials Engineering, 4(1), 6-10.

Annamalai, C., Watada, J., Broumi, S., Mishra, V. N., (2020), *Combinatorial Technique for Optimizing the combination*, The Journal of Engineering and Exact Sciences – jCEC, 6(2), 0189-0192.

Annamalai, C., (2010), *Applications of Exponential Decay and Geometric Series in Effective Medicine*, Advances in Bioscience and Biotechnology, 1 (1), 51-54.

Annamalai, C, (2019) Computation of Series of Series Using Annamalai's Computing Model, SSRN: https://ssrn.com/abstract=3408329 or http://dx.doi.org/10.2139/ssrn.3408329

Annamalai, C., (2020), *Novel Computing Technique in combinatorics*, Archive ouverte HAL. https://hal.archives-ouvertes.fr/hal-02862222.

Annamalai, C., (2020), "Optimized Computing Technique for Combination in Combinatorics, Archive ouverte HAL. https://hal.archives-ouvertes.fr/hal-02865835.

Annamalai, C., (2022), "Combinatorial Relation of Optimized Combination with Permutation", OSF Preprints, DOI: https://doi.org/10.31219/osf.io/ymk8b.

Annamalai, C., (2022), Novel Binomial Series and its Summations", *Authorea*. DOI: **10.22541/au.164933885.53684288/v2**.

Annamalai, C., (2022), Comparison between Optimized and Traditional Combinations of Combinatorics, OpenAire, DOI: 10.5281/zenodo.6462782.