OPEN ACCESS

The Journal of Engineering and Exact Sciences – jCEC, Vol. 08 N. 09 (2022)
journal homepage: https://periodicos.ufv.br/jcec
eISSN: 2527-1075
ISSN: 2446-9416

Lemma on the Binomial Coefficients of Combinatorial Geometric Series

Article Info:
Article history: Received 2022-09-28 / Accepted 2022-10-13/ Available online 2022-10-13
doi: 10.18540/jcecvl8iss9pp14760-01i

Chinnaraji Annamalai
ORCID: https://orcid.org/0000-0002-0992-2584
Department of Management, Indian Institute of Technology,
Kharagpur, West Medinipur, West Bengal, Kharagpur – 721302, India
E-mail: anna@iitkgp.ac.in
Antonio Marcos de Oliveira Siqueira
ORCID: https://orcid.org/0000-0001-9334-0394
Federal University of Viçosa, Brazil
E-mail: antonio.siqueira@ufv.br

Abstract
This paper presents lemmas and its corollaries on the combinatorial geometric series and summation of series of binomial coefficients. Also, the coefficient for each term in combinatorial geometric series refers to a binomial coefficient. These ideas can enable the scientific researchers to solve the real life problems.

Keywords: computation**,** combinatorics, binomial coefficient

1. Introduction

When the author of this article was trying to develop the multiple summations of geometric series (Annamalai, 2010, 2017a, 2017b, 2017c, 2018a, 2018b, 2018c, 2018d, 2019a, 2019b, 2020), a new idea stimulated his mind to create a combinatorial geometric series (Annamalai, 2022a, 2022b, 2022c). The combinatorial geometric series is a geometric series whose coefficient of each term of the geometric series denotes the binomial coefficient V_n^r. In this article, lemmas and corollaries on the binomial coefficients of combinatorial geometric series (Annamalai, 2022d, 2022j) are provided with detailed proofs.

2. Combinatorial Geometric Series

The combinatorial geometric series (Annamalai, 2022d, 2022e, 2022h) is derived from the multiple summations of geometric series. The coefficient of each term in the combinatorial refers to the binomial coefficient V_n^r (Annamalai, 2022i).

$$\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\sum_{i_3=i_2}^{n}\cdots\sum_{i_r=i_{r-1}}^{n} x^{i_r} = \sum_{i=0}^{n} V_i^r x^i \;\&\; V_n^r = \frac{(n+1)(n+2)(n+3)\cdots(n+r-1)(n+r)}{r!},$$

where $n \geq 0, r \geq 1$ and $n, r \in N = \{0, 1, 2, 3, \cdots\}$.

Here, $\sum_{i=0}^{n} V_i^r x^i$ refers to the combinatorial geometric series and

V_n^r is the binomial coefficient for combinatorial geometric series.

Lemma 2.1: $V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$.

Proof. Let us prove this lemma using the combinatorial geometric series.

By substituting $x = 1$ in the combinatorial geometric series $\sum_{i=0}^{n} V_i^r x^i$, we get

$$\sum_{i=0}^{n} V_i^r (1)^i = \sum_{i=0}^{n} V_i^r = V_0^r + V_1^r + V_2^n + V_3^r + \cdots + V_{n-1}^r + V_n^r = V_n^{r+1}. \quad (1)$$

This is one of the binomial identities based on the combinatorial geometric series.
From the above binomial identity, we get the following result:

$$V_{n-1}^{r+1} + V_n^r = V_n^{r+1}, \left(\because \sum_{i=0}^{n-1} V_i^r = V_{n-1}^{r+1}\right). \quad (2)$$

Let us prove the binomial equation $V_{n-1}^{r+1} + V_n^r = V_n^{r+1}$. (3)

$$V_{n-1}^{r+1} + V_n^r = \frac{n(n+1)(n+2)\cdots(n+r)}{(r+1)!} + \frac{(n+1)(n+2)\cdots(n+r)}{r!}$$
$$= \frac{(n+1)(n+2)\cdots(n+r)}{r!}\left(\frac{n}{r+1} + 1\right) =$$
$$= \frac{(n+1)(n+2)\cdots(n+r)}{r!}\left(\frac{n+r+1}{r+1}\right).$$
$$V_{n-1}^{r+1} + V_n^r = \frac{(n+1)(n+2)\cdots(n+r)(n+r+1)}{(r+1)!} = V_n^{r+1}. \quad (4)$$

Hence, the lemma is proved.

We know that the binomial series is $\sum_{i=0}^{n} V_i^{n-i} x^i y^{n-i} = (x+y)^n$.

For examples: by substituting $x = 1$ and $y = 1$ in the binomial series, we get $\sum_{i=0}^{n} V_i^{n-i} = 2^n$ and also, by substituting $x = 1$ and $y = 2$ in the binomial series, we get $\sum_{i=0}^{n} V_i^{n-i} 2^{n-i} = 3^n$.

From the binomial series given in the examples, we get the following series:

$$\sum_{i=0}^{n} V_i^{n-i}(2^{n-i} - 1) = 3^n - 2^n. \quad (5)$$

Lemma 2.2: $\sum_{i=1}^{n} V_i^{n-i} = \sum_{i=0}^{n-1} 2^i = 2^n - 1.$

Proof. Let us prove this lemma using the summation of series of binomial coefficients and sum of geometric series of powers of two as follows:

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1 \Rightarrow \sum_{i=0}^{n-1} 2^i = 2^n - 1 \text{ and } \sum_{i=0}^{n} V_i^{n-i} = 2^n \Rightarrow \sum_{i=1}^{n} V_i^{n-i} = 2^n - 1.$$

From these expressions, we conclude that

$$\therefore \sum_{i=1}^{n} V_i^{n-i} = \sum_{i=0}^{n-1} 2^i = 2^n - 1. \quad (6)$$

Hence, the lemma is proved.

Corollary 3.1: $\sum_{i=k}^{n-1} 2^i = 2^n - 2^k$. In this summation, if $k = 0$, then $\sum_{i=0}^{n-1} 2^i = 2^n - 1$.

Corollary 3.2: $\sum_{i=k+1}^{n} V_i^{n-i} = 2^n - \frac{1}{(n-k)!} \sum_{i=k}^{n-k} (k+i)$.

Corollary 3.3: $\sum_{i=0}^{r} V_i^n = V_r^{n+1} \Rightarrow \sum_{i=k+1}^{r} V_i^n = V_r^{n+1} - V_k^{n+1}$

3. Conclusion

In this article, lemmas and its corollaries were introduced on the combinatorial geometric series and series of binomial coefficients. This idea can enable the scientific researchers to solve the real-life problems (Annamalai, 2010).

References

Annamalai, C. (2010) Applications of exponential decay and geometric series in effective medicine dosage. *Advances in Bioscience and Biotechnology*, 1(1), 51-54. https://doi.org/10.4236/abb.2010.11008.

Annamalai, C. (2017a) Analysis and Modelling of Annamalai Computing Geometric Series and Summability. *Mathematical Journal of Interdisciplinary Scienc*es, 6(1), 11-15. https://doi.org/10.15415/mjis.2017.61002.

Annamalai, C. (2017b) Annamalai Computing Method for Formation of Geometric Series using in Science and Technology. *International Journal for Science and Advance Research In Technology*, 3(8), 187-289. http://ijsart.com/Home/IssueDetail/17257.

Annamalai, C. (2017c) Computational modelling for the formation of geometric series using Annamalai computing method. *Jñānābha*, 47(2), 327-330.https://zbmath.org/?q=an%3A1391.65005.

Annamalai, C. (2018a) Algorithmic Computation of Annamalai's Geometric Series and Summability. *Journal of Mathematics and Computer Science*, 3(5),100-101. https://doi.org/10.11648/j.mcs.20180305.11.

Annamalai, C. (2018b) Annamalai's Computing Model for Algorithmic Geometric Series and Its Mathematical Structures. *Journal of Mathematics and Computer Science*, 3(1),1-6 https://doi.org/10.11648/j.mcs.20180301.11.

Annamalai, C. (2018c) Computing for Development of a New Summability on Multiple Geometric Series. *International Journal of Mathematics, Game Theory and Algebra*, 27(4), 511-513.

Annamalai, C. (2018d) Novel Computation of Algorithmic Geometric Series and Summability. *Journal of Algorithms and Computation*, 50(1), 151-153. https://www.doi.org/10.22059/JAC.2018.68866.

Annamalai, C. (2019a) A Model of Iterative Computations for Recursive Summability. *Tamsui Oxford Journal of Information and Mathematical Sciences*, 32(1), 75-77.

Annamalai, C. (2019b) Recursive Computations and Differential and Integral Equations for Summability of Binomial Coefficients with Combinatorial Expressions. International Journal of Scientific Research in Mechanical and Materials Engineering, 4(1), 6-10. https://ijsrmme.com/IJSRMME19362.

Annamalai, C. (2020) Combinatorial Technique for Optimizing the combination. *The Journal of Engineering and Exact Sciences*, 6(2), 0189-0192. https://doi.org/10.18540/jcecvl6iss2pp0189-0192.

Annamalai, C. (2022a) Computation and Calculus for Combinatorial Geometric Series and Binomial Identities and Expansions. *The Journal of Engineering and Exact Sciences*, 8(7), 14648–01i. https://doi.org/10.18540/jcecvl8iss7pp14648-01i.

Annamalai, C. (2022b) Application of Factorial and Binomial identities in Information, Cybersecurity and Machine Learning. International Journal of Advanced Networking and Applications, 14(1), 5258-5260. https://doi.org/10.33774/coe-2022-pnx53-v21.

Annamalai, C. (2022c) Combinatorial and Multinomial Coefficients and its Computing Techniques for Machine Learning and Cybersecurity. *The Journal of Engineering and Exact Sciences*, 8(8), 14713–01i. https://doi.org/10.18540/jcecvl8iss8pp14713-01i.

Annamalai, C. (2022d) Annamalai's Binomial Identity and Theorem, *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4097907.

Annamalai, C. (2022e) Computing Method for Combinatorial Geometric Series and Binomial Expansion. *SSRN Electronic Journal*. http://dx.doi.org/10.2139/ssrn.4168016.

Annamalai, C. (2022f) Computation of Multinomial and Factorial Theorems for Cryptography and Machine Learning. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks-v9.

Annamalai, C. (2022g) Computation of Binomial, Factorial and Multinomial Theorems for Machine Leaning and Cybersecurity. *COE, Cambridge University Press*. https://doi.org/10.33774/coe-2022-b6mks-v11.

Annamalai, C. (2022h) Series and Summations on Binomial Coefficients of Optimized Combination. *The Journal of Engineering and Exact Sciences*, 8(3), 14123-01e. https://doi.org/10.18540/jcecvl8iss3pp14123-01e.

Annamalai, C. (2022i) Factorials and Integers for Applications in Computing and Cryptography. *COE, Cambridge University Press.* https://doi.org/10.33774/coe-2022-b6mks.

Annamalai, C. (2022j) Multinomial Theorem on the Binomial Coefficients for Combinatorial Geometric Series. *SSRN Electronic Journal.* http://dx.doi.org/10.2139/ssrn.4202632.