

Prediction of Thermal Distribution in an Internally Heated Radiative-Convective Moving Porous Fin with variable Thermal Conductivity using Homotopy Perturbation Method

Article Info:

Article history: Received 2022-10-02 / Accepted 2022-11-02/ Available online 2022-11-02

doi: 10.18540/jcecv18iss8pp14923-01i



Suraju A. Oladosu

Department of Mechanical Engineering, Faculty of Engineering, Lagos State University, Epe Campus, Lagos, Nigeria

E-mail: suraju.oladosu@lasu.edu.ng

Rafiu Olalekan Kuku

Department of Mechanical Engineering, Faculty of Engineering, Lagos State University, Epe Campus, Lagos, Nigeria

E-mail: rafiu.kuku@lasu.edu.ng

Gbeminiyi Musibau Sobamowo

ORCID: <https://orcid.org/0000-0003-2402-1423>

Department of Mechanical Engineering, Faculty of Engineering, University of Lagos, Akoka Lagos, Nigeria

E-mail: gsobamowo@unilag.edu.ng

Antonio Marcos de Oliveira Siqueira

ORCID: <https://orcid.org/0000-0001-9334-0394>

Federal University of Viçosa, Brazil

E-mail: antonio.siqueira@ufv.br

Abstract

Thermal distribution of a rectangular moving convective-radiative porous fin with temperature-dependent thermal conductivity and internal heat generation is analyzed in this work homotopy perturbation method. With the aid of the approximate analytical solutions, the impacts of the model parameters on the thermal behaviour of the fin are investigated. The parametric analysis reveals that increase in porosity and convective parameters, thermal distribution decreases. The values of the temperature distribution in the fin increase as the Peclet number increases. Also, as the thermal conductivity and internal heat generation increase, the fin thermal distribution increases. It is believed that the present study will help in the device of the thermal passive devices.

Keywords: Thermal analysis; Porous Fin; Convective fin; Moving fin; Variable thermal conductivity; homotopy perturbation method.

Nomenclature

- A Cross sectional area of the fins, (m^2).
 h Heat transfer coefficient, ($Wm^{-2}k^{-1}$).
 C_p Specific heat of the fluid passing through porous fin ($J/kg\cdot K$).
 Da Darcy number.
 g Gravity constant(m/s^2).
 h Heat transfer coefficient over the fin surface (W/m^2K).
 H Dimensionless heat transfer coefficient at the base of the fin, ($Wm^{-2}k^{-1}$).
 k Thermal conductivity of the fin material, ($Wm^{-1}k^{-1}$).
 k_{eff} Effective thermal conductivity of the porous fin.
 k_0 Permeability of the porous fin (m^2).
 b Length of the fin, (m).
 M Dimensionless Convective parameter.
 \dot{m} Mass flow rate of fluid passing through porous fin (kg/s).
 p Perimeter of the fin (m).
 Pe Peclet number.
 Q Dimensionless heat transfer rate per unit area.
 q internal heat generation
 q_T Heat flux.
 Sp Porosity parameter.
 t^* Thickness of the fin.
 T Fin temperature (K).
 T_s Ambient temperature, (K).
 T_b Temperature at the base of the fin, (K).
 U Velocity of fin (m/s).
 v_w velocity of fluid passing through the fin at any point (m/s).
 w Width of the fin (m).
 x Axial length measured from fin tip (m).
 X Dimensionless axial length of the fin.
 H_t Dimensionless internal heat generation parameter
 G Heat Generation number

Greek Symbols

- β Thermal conductivity parameter or non-linear parameter.
 θ Dimensionless temperature.
 η Efficiency of the fin.
 ν Kinematic viscosity(m^2/s).
 ρ Density of the fluid(kg/m^3).
 λ Measure of thermal conductivity variation with temperature.

Subscripts

- s Solid properties.
 f Fluid properties.
 eff Effective porous properties.

1. Introduction

The applications of passive devices such as fins and spines have been widely used for heat transfer augmentations in thermal and electronics systems. Also, the study of thermal behavior of continuous moving surfaces such as extrusion, hot rolling, glass sheet or wire drawing, casting, powder metallurgy techniques for the fabrication of rod and sheet has become an area of increasing research interests. In the processes such as rolling of strip, hot rolling, glass fiber drawing, casting, extrusion, drawing of wires and sheets, there is an exchange of heat between material and the surroundings while the material moves through the roller of or furnace. Since, the operations and the thermal configuration of these metal fabrication technologies satisfy the criterion for fin approximation, they can be modeled as continuously moving fins. Due to these adaptable and wide applications, there have extensive research on continuous moving fins.

Moreover, in industrial processes, control of cooling rate of the sheets is very important to obtain desired material structure. Consequently, various studies on heat transfer and thermal analysis of a continuously moving fin have been carried out by many researchers. Torabi *et al.* (2012) presented the analytical solutions for convective-radiative heat transfer in a continuously moving fin with variable thermal conductivity while Aziz & Lopez (2011) studied the convection–radiation heat transfer from a continuously moving surface with variable thermal conductivity. Aziz and Khani (2011) adopted homotopy analysis method to analyze the same problem. Singh *et al.* (2013) carried the thermal analysis of a convective-radiative continuously moving fin with temperature-dependent thermal conductivity using wavelet collocation method. Also, Aziz & Torabi (2012) analyzed the moving fin with temperature-dependent thermal properties.

The effectiveness of spectral element method was demonstrated in the study carried out by Ma *et al.* (2016) on thermal behaviour of longitudinal porous moving fins of different profiles. Sun *et al.* (2015) predicted the heat transfer in convective-radiative moving rod using spectral collocation method while Aziz and Khani (2011) adopted homotopy analysis method to analyze the same problem. In the previously cited work of Aziz and Lopez (2011), fourth-fifth-order Runge-Kutta-Fehlberg method was used to numerically investigate of the convective-radiative moving fin while Torabi *et al.* (2012) utilized differential transformation method for the continuously moving fin losing heat through both convection and radiation and having temperature-dependent thermal conductivity.

In another study, Ravi Kanth & Kumar (2013) applied Haar wavelet method for the moving convective-radiative Fin with variable thermal conductivity. Singla & Das (2014) used Adomian decomposition method for the inverse heat transfer problems in a moving fin. Moradi & Rafiee (2013) developed analytical solution using differential transformation method for the convection-radiation heat transfer in a continuously moving fin with temperature-dependent thermal conductivity. Dogonchi & Ganji (2016) considered the flow of convection-radiation heat transfer in moving fin with variable thermal properties and internal heat generation while Sun & Ma (2015) used collocation spectral method to theoretically investigate the same problem. Singh *et al.* (2013) used wavelet collocation approach for convective-radiative moving fin with variable thermal Conductivity. With the application of simplex search method, Das (2011) presented for a conductive-convective fin with variable conductivity.

Apart from the fact that there are very few studies on thermal analysis of moving porous fin, to the best of the authors' knowledge, comparative and predictive studies of heat transfer in a moving convective-radiative porous fin with temperature-dependent thermal conductivity and internal heat generation using differential transformation and finite difference methods have not been carried out. Therefore, Thermal distribution of a rectangular moving convective-radiative porous fin with temperature-dependent thermal conductivity and internal heat generation is analyzed in this work homotopy perturbation method. With the aid of the analytical solutions, the impacts of the model parameters on the thermal behaviour of the fin are investigated.

2. Problem formulation

It is considered as shown in Figure 1 that a straight rectangular porous fin of constant cross-sectional area, A , with length, b , width W , and thickness, t^* is subjected to heat from the prime surface. The fin is porous to allow the flow of fluid infiltrate through it. The axial coordinate, x is measured from the base of the fin. The fin is moving horizontally at a constant velocity U_x . The development of the thermal model is based on the the assumptions that

- i. the porous medium in the porous fin is homogeneous and isotropic and it is saturated with single-phase fluid.
- ii. There exists thermal equilibrium between the fluid and the solid matrix of the porous fin.
- iii. The fin is in perfect contact with the prime surface.
- iv. The heat loss through the tip of the fin is negligible.
- v. With the exception of thermal conductivity of the fin, all the other thermal and physical properties of the fin are assumed constant.

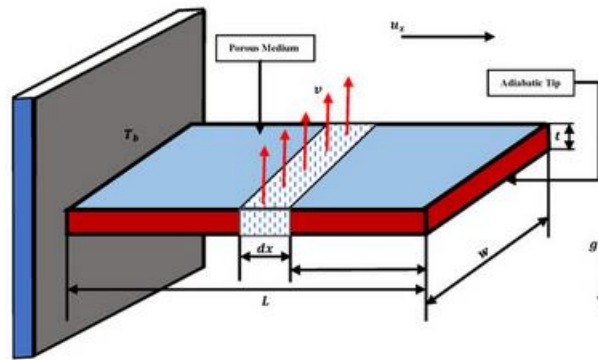


Figure 1 Schematic of the convective longitudinal porous fin

Based on energy balance and Darcy formulation for the porous medium, the thermal model of the moving fin is developed as

$$\frac{d}{dx} \left(k(T) \frac{\partial T}{\partial x} \right) - \frac{\rho C_p g k \beta^* (T - T_a)^2}{vt} - \frac{hP(T - T_a)}{A} - \frac{\sigma \in P(T^4 - T_a^4)}{A} - \frac{\rho c_p V_w}{A} \frac{dT}{dx} + \frac{q_o^*(T)}{A} = 0 \quad (1)$$

The thermal conductivity and internal heat generation vary linearly with temperature as

$$k(T) = k_0 (1 + \lambda (T - T_s)) \quad (2)$$

$$q^*(T) = q_o^* (1 + \varepsilon (T - T_s)) \quad (3)$$

Substituting equations (3)-(4) into equation (1), gives

$$\frac{d}{dx} \left((1 + \lambda(T - T_s)) \frac{\partial T}{\partial x} \right) - \frac{\rho C_p g k \beta^* (T - T_a)^2}{\nu k_o t} - \frac{hP(T - T_a)}{k_o A} - \frac{\sigma \in P(T^4 - T_a^4)}{A} - \frac{\rho c_p u}{k_o A} \frac{dT}{dx} + \frac{q_o^* (1 + \varepsilon(T - T_a))}{k_o A} = 0 \quad (4)$$

Subject to the following boundary conditions;

$$\text{when } x = 0, \quad T = T_b \quad (5)$$

$$\text{when } x = L, \quad \frac{dT}{dx} = 0 \quad (6)$$

when there is small temperature difference within the material/fin during the heat flow, the term T^4 can be expressed as a linear function of temperature. Therefore, we have

$$T^4 = T_a^4 + 4T_a^3(T - T_a) + 6T_a^2(T - T_a)^2 + \dots \cong 4T_a^3T - 3T_a^4 \quad (7)$$

$$\frac{d}{dx} \left((1 + \lambda(T - T_s)) \frac{\partial T}{\partial x} \right) - \frac{\rho C_p g k \beta^* (T - T_a)^2}{\nu k_o t} - \frac{hP(T - T_a)}{k_o A} - \frac{4\sigma \in T_a^3(T - T_a)}{k_a \delta} - \frac{\rho c_p u}{k_o A} \frac{dT}{dx} + \frac{q_o^* (1 + \varepsilon(T - T_a))}{k_o A} = 0 \quad (8)$$

Introducing the following dimensionless variables,

$$\theta = \frac{T - T_s}{T_b - T_s} \quad X = \frac{x}{L}, \quad N_C^2 = \frac{hP L^2}{k_o A} \quad S_h = \frac{g k \beta^* b^2 (T_b - T_s)}{\nu \alpha t} = \frac{DaxRa}{K} \left(\frac{L}{t} \right)^2, \quad N_R = \frac{4\sigma \in P T_a^3 L^2}{k_a A} \quad (9)$$

$$\alpha = \frac{k_o}{\rho C_p} \quad Pe = \frac{UL}{\alpha} \quad \beta = \lambda(T_b - T_s) \quad \gamma = \varepsilon(T_b - T_s), \quad G = \frac{q_o^*}{hP(T_b - T_s)}, \quad M^2 = N_C + N_R$$

where M is convection-radiative parameter that indicates the effect of surface convection of the fin; S_p is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect, β is the dimensionless thermal conductivity and Pe is the Peclet number which represent the dimensionless speed of the moving fin ($Pe = 0$ represents a stationary fin). G represents the heat generation number. H_i , represents the dimensionless internal heat generation number.

Substituting the dimensionless variables into equation (10), gives the non-dimensional governing equation

$$\frac{d^2 \theta}{dX^2} + \beta \theta \frac{d^2 \theta}{dX^2} + \beta \left(\frac{d\theta}{dX} \right)^2 - S_h \theta^2 - M^2 \theta - Pe \frac{d\theta}{dX} + M^2 G (1 + \gamma \theta) = 0 \quad (10)$$

The boundary conditions appear in dimensionless form as

$$\text{when } X = 0, \quad \theta = 1 \quad (11)$$

$$\text{when } X = 1, \quad \frac{d\theta}{dx} = 0 \quad (12)$$

3. Method of Solution by homotopy Perturbation Method

Due to the nonlinear terms in Equation (10), the development of exact analytical solution will be difficult. Therefore, in this work, homotopy perturbation method is used to solve the equation.

3.1 The basic idea of homotopy perturbation method

In order to establish the basic idea behind homotopy perturbation method, consider a system of nonlinear differential equations given as

$$A(U) - f(r) = 0, \quad r \in \Omega \quad (13)$$

with the boundary conditions

$$B\left(u, \frac{\partial u}{\partial \eta}\right) = 0, \quad r \in \Gamma \quad (14)$$

where A is a general differential operator, B is a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω

The operator A can be divided into two parts, which are L and N , where L is a linear operator, N is a non-linear operator. Equation (13) can be therefore rewritten as follows

$$L(u) + N(u) - f(r) = 0 \quad (15)$$

By the homotopy technique, a homotopy $U(r, p) : \Omega \times [0, 1] \rightarrow R$ can be constructed, which satisfies

$$H(U, p) = (1-p)[L(U) - L(U_o)] + p[A(U) - f(r)] = 0, \quad p \in [0, 1] \quad (16)$$

Or

$$H(U, p) = L(U) - L(U_o) + pL(U_o) + p[N(U) - f(r)] = 0 \quad (17)$$

In the above Equation (16) and (17), $p \in [0, 1]$ is an embedding parameter, u_o is an initial approximation of equation of Equation (13), which satisfies the boundary conditions.

Also, from Equation (16) and (17), we will have

$$H(U, 0) = L(U) - L(U_o) = 0 \tag{18}$$

$$H(U, 0) = A(U) - f(r) = 0 \tag{19}$$

The changing process of p from zero to unity is just that of $U(r, p)$ from $u_o(r)$ to $u(r)$. This is referred to homotopy in topology. Using the embedding parameter p as a small parameter, the solution of Equation (16) and (17) can be assumed to be written as a power series in p as given in Equation (20):

$$U = U_o + pU_1 + p^2U_2 + \dots \tag{20}$$

It should be pointed out that of all the values of p between 0 and 1, $p=1$ produces the best result. Therefore, setting $p=1$, results in the approximation solution of Equation (13)

$$u = \lim_{p \rightarrow 1} U = U_o + U_1 + U_2 + \dots \tag{21}$$

The basic idea expressed above is a combination of homotopy and perturbation method. Hence, the method is called homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantages of the traditional perturbation techniques. The series Equation (21) is convergent for most cases.

3.2 Application of the homotopy perturbation method to the present problem

According to homotopy perturbation method (HPM), one can construct an homotopy for Equation (10) as

$$H(\theta, p) = (1-p) \left[\frac{d^2\theta}{dX^2} \right] + p \left[\frac{d^2\theta}{dX^2} + \beta\theta \frac{d^2\theta}{dX^2} + \beta \left(\frac{d\theta}{dX} \right)^2 - S_n\theta^2 - M^2\theta - Pe \frac{d\theta}{dX} + M^2G(1+\gamma\theta) \right] = 0 \tag{22}$$

where $p \in [0, 1]$ is an embedding parameter. For $p=0$ and $p=1$ we have

$$\theta(X, 0) = \theta_0(X) \quad , \quad \theta(X, 1) = \theta_0(X) \tag{23}$$

Note that when p increases from 0 to 1, $\theta(X, p)$ varies from $\theta_0(X)$ to $\theta_0(X)$.

Supposing that the solution of Equation (10) can be expressed in a series in p :

$$\theta(X) = \theta_0(X) + p\theta_1(X) + p^2\theta_2(X) + p^3\theta_3(X) + \dots = \sum_{i=0}^n p^i \theta_i(X) \tag{24}$$

On substituting Equation (24) and into Equation (22) and expanding the equation and collecting all terms with the same order of p together, the resulting equation appears in form of polynomial in p . On equating each coefficient of the resulting polynomial in p to zero, we arrived at a set of differential equations and the corresponding boundary conditions as

$$p^0 : \frac{d^2\theta_0}{dX^2} = 0, \tag{25}$$

$$\text{The boundary conditions are } \theta_0(0) = 1, \theta'_0(1) = 0, \tag{26}$$

$$p^1 : \frac{d^2\theta_1}{dX^2} + \beta\theta_0 \frac{d^2\theta_0}{dX^2} + \beta \left(\frac{d\theta_0}{dX} \right)^2 - S_h\theta_0^2 - M^2\theta_0 - Pe \frac{d\theta_0}{dX} + M^2G(1 + \gamma\theta_0) = 0, \tag{27}$$

$$\text{The boundary conditions are } \theta_1(0) = 0, \theta'_1(1) = 0, \tag{28}$$

$$p^2 : \frac{d^2\theta_2}{dX^2} + \beta\theta_1 \frac{d^2\theta_0}{dX^2} + \beta\theta_0 \frac{d^2\theta_1}{dX^2} + 2\beta \left(\frac{d\theta_0}{dX} \frac{d\theta_1}{dX} \right) - 2S_h\theta_0\theta_1 - M^2\theta_1 - Pe \frac{d\theta_1}{dX} + M^2G\gamma\theta_1 = 0, \tag{29}$$

$$\text{The boundary conditions are } \theta_2(0) = 0, \theta'_2(1) = 0, \tag{30}$$

$$p^3 : \frac{d^2\theta_3}{dX^2} + \beta\theta_0 \frac{d^2\theta_2}{dX^2} + \beta\theta_1 \frac{d^2\theta_1}{dX^2} + \beta\theta_2 \frac{d^2\theta_0}{dX^2} + \beta \left(\frac{d\theta_1}{dX} \right)^2 + 2\beta \left(\frac{d\theta_0}{dX} \frac{d\theta_2}{dX} \right) - S_h\theta_1^2 - 2S_h\theta_0\theta_2 - M^2\theta_2 - Pe \frac{d\theta_2}{dX} + M^2G\gamma\theta_2 = 0, \tag{31}$$

$$\text{The boundary conditions are } \theta_3(0) = 0, \theta'_3(1) = 0, \tag{32}$$

On solving the above solution, we have

$$\theta_0(X) = 1; \tag{33}$$

$$\theta_1(X) = \left[\frac{M^2 + S_h - M^2G(1 + \gamma)}{2} \right] (X^2 - 2X); \tag{34}$$

$$\begin{aligned} \theta_2(X) = & 2S_h \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) - \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \beta X^2 + M^2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) \\ & + 2Pe \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^2}{2} - X \right) - M^2G\gamma \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) \\ & + \left(2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \beta + \frac{16}{3} S_h \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \right. \\ & \left. + \frac{8}{3} M^2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] - \frac{8}{3} M^2G\gamma \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \right) X \end{aligned} \tag{35}$$

From the definitions of HPM in Equation (21) we obtain the infinite series solution given by

$$\theta(X) = \theta(0) + \theta(1) + \theta(2) + \dots \tag{36}$$

Therefore, substituting Equations (33), (34) and (35) into Equation (36), we have

$$\begin{aligned} \theta(X) = & 1 + \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] (X^2 - 2X) + 2S_h \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) - \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \beta X^2 \\ & + M^2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) + 2Pe \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^2}{2} - X \right) - M^2G\gamma \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \left(\frac{X^4}{12} - \frac{X^3}{3} \right) \\ & + \left(2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \beta + \frac{16}{3} S_h \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \right. \\ & \left. + \frac{8}{3} M^2 \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] - \frac{8}{3} M^2G\gamma \left[\frac{M^2 + S_h - M^2G(1+\gamma)}{2} \right] \right) X; \end{aligned} \tag{37}$$

4. Exact analytical solutions

If the nonlinear terms are removed, the dimensionless form of the governing equation reduces to

$$\frac{\partial^2 \theta}{\partial X^2} - Pe \frac{\partial \theta}{\partial X} - M^2(1 - G\gamma)\theta = -M^2G \tag{38}$$

And the exact analytical solution is given by

$$\theta(X) = \left\{ \frac{\left[Pe + \sqrt{Pe^2 + 4M^2(1-G\gamma)} \right] \left[\frac{G}{(1-G\gamma)} - 1 \right] \left[e^{-\left(\frac{Pe + \sqrt{Pe^2 + 4M^2(1-G\gamma)}}{2} \right) X} + e^{-\left(\frac{Pe - \sqrt{Pe^2 + 4M^2(1-G\gamma)}}{2} \right) X} \right]}{\left[Pe - \sqrt{Pe^2 + 4M^2(1-G\gamma)} \right] e^{-\left(\frac{Pe + \sqrt{Pe^2 + 4M^2(1-G\gamma)}}{2} \right) X} - \left[Pe + \sqrt{Pe^2 + 4M^2(1-G\gamma)} \right] e^{-\left(\frac{Pe - \sqrt{Pe^2 + 4M^2(1-G\gamma)}}{2} \right) X}} \right\} + \frac{G}{(1-G\gamma)} \tag{39}$$

5. Results and Discussion

With aid of the MATLAB, the symbolic solutions are simulated, and the results are given in Figure 3-8. These results show the parametric studies of the effects of the model parameters on the thermal behaviour of the extended surface. However, prior to this parametric study, it is required to verify the results of the present approximate analytical solutions. The results of the developed exact analytical solutions are compared with the approximate analytical solutions i.e., homotopy perturbation method used in the present study. Table 1 shows the comparison of the results. The Table depict that there are excellent agreements between the results of the two analytical methods for the linear thermal model. However, the approximate analytical solution provides insight into the nonlinear thermal behaviour of the device.

Table 1: Comparison of results of linear thermal model $\theta(X)$ for $Sp=0; \beta=0; Pe=0.1; M=0.1; Ht=0.0, G=0.0$:

	Exact	HPM
1.00	0.994851	0.994851
0.90	0.994091	0.994091
0.80	0.995052	0.995052
0.70	0.995304	0.995304
0.60	0.995658	0.995658
0.50	0.996116	0.996116
0.40	0.996679	0.996679
0.30	0.997348	0.997348
0.20	0.998123	0.998123
0.10	0.999007	0.999007
0.00	1.000000	1.000000

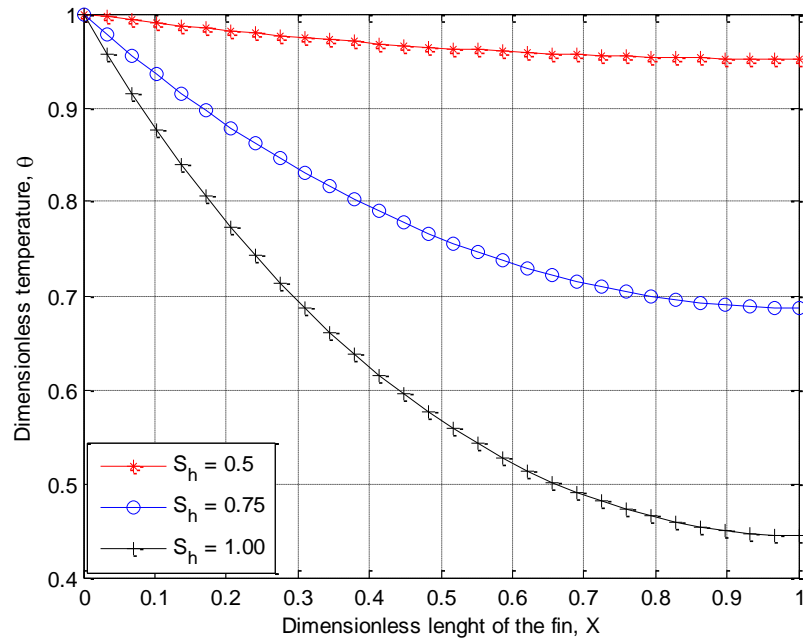


Figure 2 Effect of porous term on the dimensionless temperature distribution of the fin

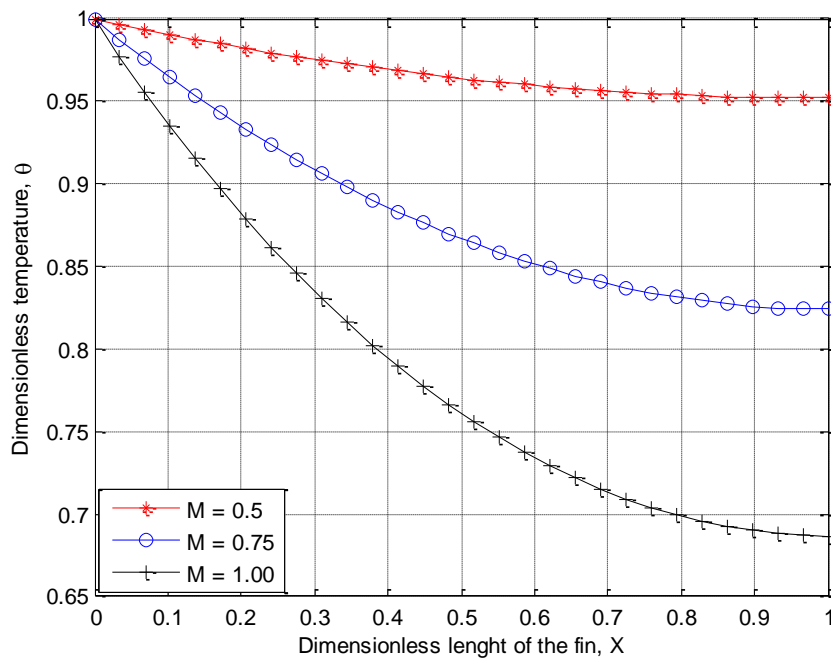


Figure 3 Effect of convective-term term on the dimensionless temperature distribution of the fin

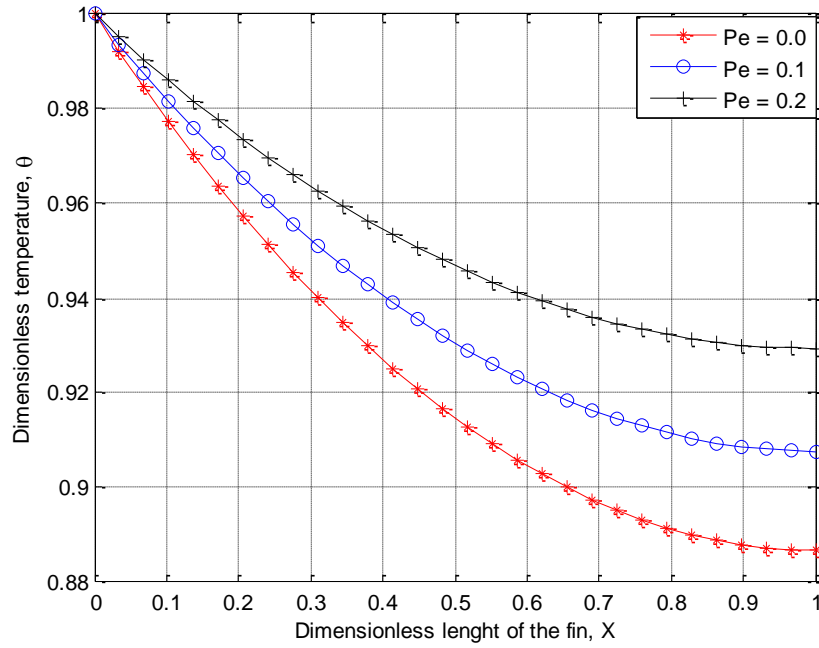


Figure 4 Effect of Peclet number on the dimensionless temperature distribution of the fin

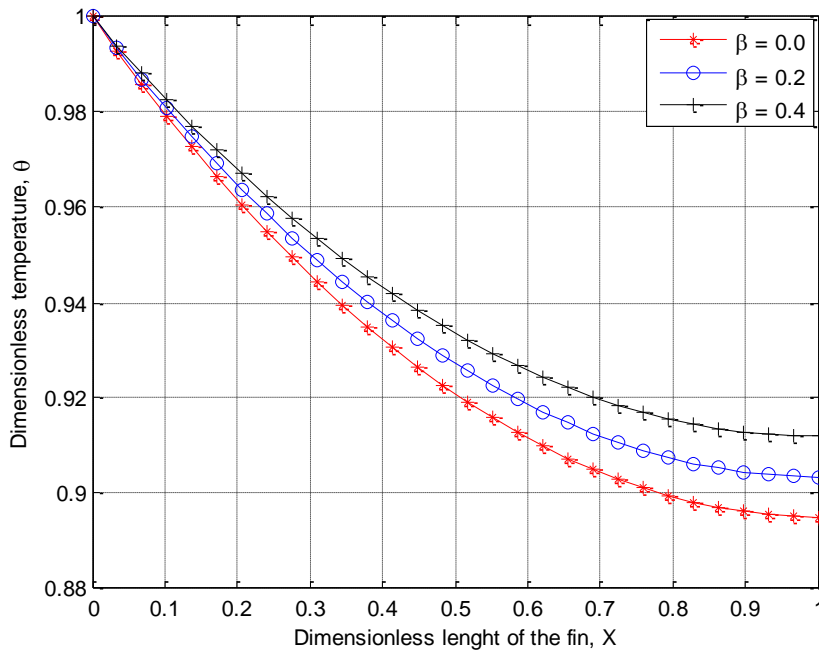


Figure 5 Effect of nonlinear thermal conductivity term on the dimensionless temperature distribution of the fin

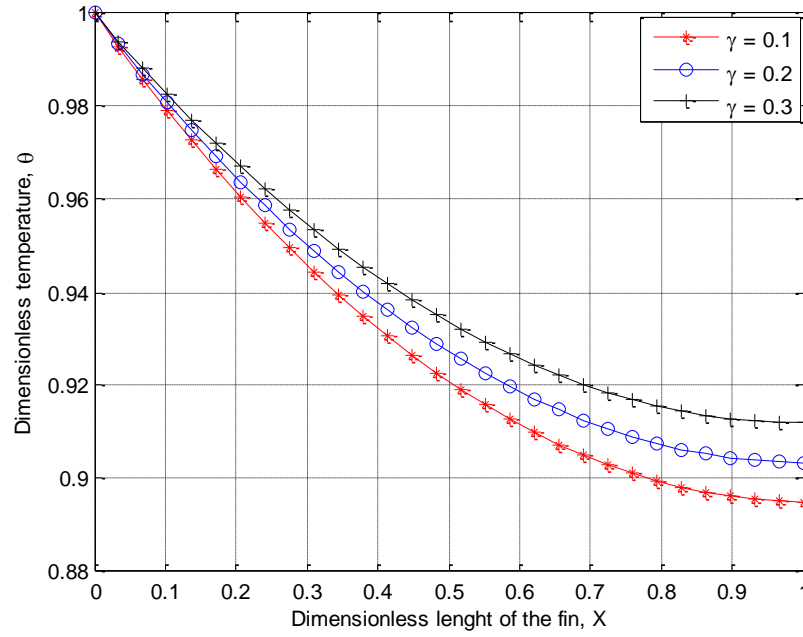


Figure 6 Effect of temperature-variant internal heat generation term on the dimensionless temperature distribution of the fin

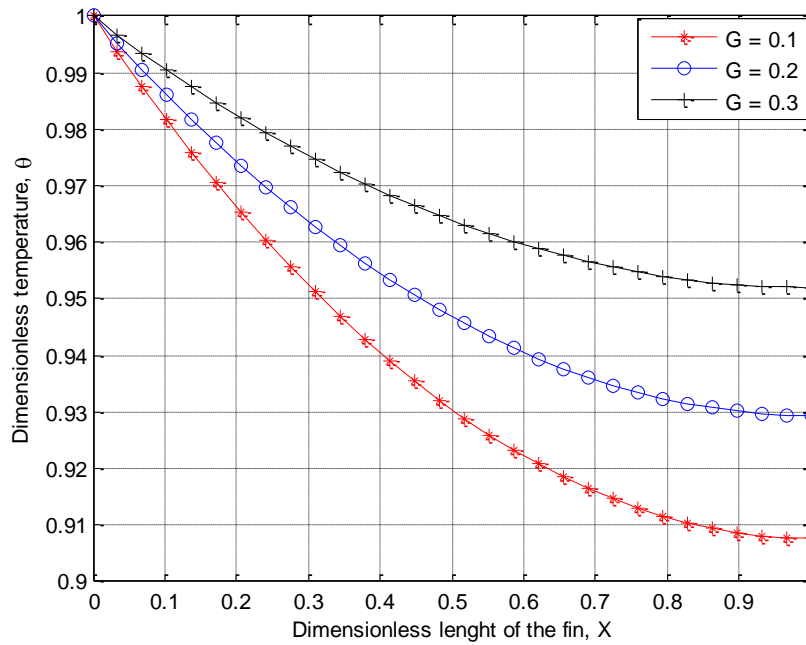


Figure 7 Effect of internal heat generation term on the dimensionless temperature distribution of the fin

Figure 2 shows the effects of porous parameter or porosity on the temperature distribution in the porous fin are shown. From the figures, as the porosity parameter increases, the temperature decreases rapidly and the rate of heat transfer (the convective-radiative heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. The rapid decrease in fin temperature due to increase in the porosity parameter is because as porosity parameter increases, the permeability of the porous fin increases and therefore the ability of the working fluid to penetrate through the fin pores increases, the effect of buoyancy force increases and thus the fin convects more heat, the rate of heat transfer from the fin is enhanced and the thermal performance of the fin is increased. High value of porosity parameter not only decreases the effective thermal conductivity but also reduces ideal heat transfer.

The impacts of the thermal-geometric variable, M , on the temperature distribution in the fin is shown Figure 3. It is illustrated that the temperature of the fin increases when the value of M decreases. It is evident that as the convection-conduction parameter is increased it contributes to more heat loss from the fin and hence cooling of the fin occurs which shows a decrease in the temperature profile.

Figure 4 shows the influence of Peclet number on the fin temperature profiles. It is shown that the fin temperature increases with an increase in the Peclet number. This is because when the Peclet number increases, it is an indication that the fin moves faster and the time for which the material is exposed to the environment gets shorter as well as the losing heat form fin surface gets stronger, thus the fin temperature increases.

Figure 5 establishes the significance of the thermal conductivity parameter β on the temperature distribution along the fin. It is shown that when the thermal conductivity parameter is increased, the temperature of the fin increases because of the enhancement of the heat conduction process which leads to an increase in the local temperature of the fin. The impact of internal heat generation on the thermal behaviour of the fin is shown in Figures 6 and 7. It is shown that the fin temperature increases as the internal heat generation increases.

5. Conclusion

In this paper, thermal distribution of a rectangular moving convective-radiative porous fin with temperature-dependent thermal conductivity and internal heat generation is analyzed in this work homotopy perturbation method. Also, from the analysis, it is found that

- i. increase in porosity and convective parameters, the rate of heat transfer from the fin increases and consequently improve the efficiency of the fin.
- ii. the thermal efficiency of the fin is improved by increasing the Peclet number.
- iii. as thermal conductivity and the internal heat generation increase, the rate of heat transfer from the fin decreases.
- iv. increase in thermal conductivity and the internal heat generation cause the rate of heat transfer from the fin to decrease.

It is believed that the present study will help in the design of the passive device so as to optimal performance of the ultimate purpose of heat transfer enhancement.

References

- Aziz, A., & Khani, F. (2011). Convection–radiation from a continuously moving fin of variable thermal conductivity. *Journal of the Franklin Institute*, 348(4), 640-651.
- Aziz, A., & Khani, F. (2011). Convection–radiation from a continuously moving fin of variable thermal conductivity. *Journal of the Franklin Institute*, 348(4), 640-651.
- Aziz, A., & Lopez, R. J. (2011). Convection-radiation from a continuously moving, variable thermal conductivity sheet or rod undergoing thermal processing. *International journal of thermal sciences*, 50(8), 1523-1531.
- Aziz, A., & Torabi, M. (2012). Convective-radiative fins with simultaneous variation of thermal conductivity, heat transfer coefficient, and surface emissivity with temperature. *Heat Transfer—Asian Research*, 41(2), 99-113.
- Das, R. (2011). A simplex search method for a conductive–convective fin with variable conductivity. *International Journal of Heat and Mass Transfer*, 54(23-24), 5001-5009.
- Dogonchi, A. S., & Ganji, D. D. (2016). Convection–radiation heat transfer study of moving fin with temperature-dependent thermal conductivity, heat transfer coefficient and heat generation. *Applied thermal engineering*, 103, 705-712.
- Ma, J., Sun, Y., Li, B., & Chen, H. (2016). Spectral collocation method for radiative–conductive porous fin with temperature dependent properties. *Energy Conversion and Management*, 111, 279-288.
- Moradi, A., & Rafiee, R. (2013). Analytical solution to convection-radiation of a continuously moving fin with temperature-dependent thermal conductivity. *Thermal Science*, 17(4), 1049-1060.
- Ravi Kanth, A. S. V., & Uday Kumar, N. (2013). Application of the Haar wavelet method on a continuously moving convective-radiative fin with variable thermal conductivity. *Heat Transfer—Asian Research*, 42(4), 335-351.
- Singh, S., Kumar, D., & Rai, K. N. (2013). Wavelet Collocation Solution for Convective-Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity. *International Journal of Engineering and Advanced Technology*, 2(4), 10-16.
- Singh, S., Kumar, D., & Rai, K. N. (2013). Wavelet Collocation Solution for Convective-Radiative Continuously Moving Fin with Temperature-Dependent Thermal Conductivity. *International Journal of Engineering and Advanced Technology*, 2(4), 10-16.
- Singla, R. K., & Das, R. (2014). Application of decomposition method and inverse prediction of parameters in a moving fin. *Energy Conversion and Management*, 84, 268-281.
- Sun, Y. S., Ma, J., & Li, B. W. (2015). Spectral collocation method for convective–radiative transfer of a moving rod with variable thermal conductivity. *International Journal of Thermal Sciences*, 90, 187-196.
- Sun, Y., & Ma, J. (2015). Application of collocation spectral method to solve a convective-radiative longitudinal fin with temperature dependent internal heat generation, thermal conductivity and heat transfer coefficient. *Journal of Computational and Theoretical Nanoscience*, 12(9), 2851-2860.
- Torabi, M., Yaghoobi, H., & Aziz, A. (2012). Analytical solution for convective–radiative continuously moving fin with temperature-dependent thermal conductivity. *International Journal of Thermophysics*, 33(5), 924-941.

Torabi, M., Yaghoobi, H., & Aziz, A. (2012). Analytical solution for convective–radiative continuously moving fin with temperature-dependent thermal conductivity. *International Journal of Thermophysics*, 33(5), 924-941.