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Dynamic analysis of a translating five-bar linkage mechanism

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Abstract. In this paper, the dynamic analysis of a translating five-bar linkage mechanism is presented. The dynamic model of the mechanism is developed with the applied force to the crank arm resolved into the two principal planes, that is, x- and y- directions and the resulting reaction forces and moments at the pin joints determined at various translating acceleration of 0, 10, 20, 30 and 40 m/s^2 in the direction as well as in the opposite direction of the crank rotation. It was observed that the horizontal reaction forces at the pin joints were decreasing as the translating acceleration increases in the direction of crank rotation while it increases when the acceleration increases in the opposite direction to the crank rotation. It was also observed that the pin joint moments at point A decreases as the translation velocities increases in the crank arm rotation direction while it increases as the acceleration increases in the opposite direction. The converse is the case for pin joint B, while there were no significant differences for pin joint moments at pin joints C and D in all the considered acceleration in both directions. Also, the vertical reaction forces at the pin joints did not change in magnitude as the magnitude and directions of the acceleration were altered. The analysis brings to the fore the importance of the consideration of translating acceleration in the design, development and utilization of five – bar linkage to avoid failure at any of the joints.
Keywords: Dynamic analysis. Five – bar linkage mechanism. Pin joint moments. Pin joint reaction. Translating velocity.

Nomenclature

a_1, a_2, a_3, a_4 distances of CG from the pin end of linkages *AB*, *BC*, *CD*, and *DO*, respectively
A_x, B_x, C_x, D_x, O_x reaction forces at the pin joint *A*, *B*, *C*, *D*, *and O*, respectively in the *x* direction
A_y, B_y, C_y, D_y, O_y reaction forces at the pin joint *A*, *B*, *C*, *D*, *and O*, respectively in the *y* direction
F_r applied resultant force to the system
F_x, F_y resolution of applied force in the horizontal and vertical directions, respectively
g acceleration due to gravity
$I_{ABCG}, I_{BCCG}, I_{CDCG}, I_{DOCG}$ Moments of inertia of linkage *AB*, *BC*, *CD* and *DO*, respectively
m_1, m_2, m_3, m_4 masses of linkages *AB*, *BC*, *CD*, *and DO*, respectively
M_a, M_b, M_c, M_d, M_o moments about pin joints *A*, *B*, *C*, *D* and *O*, respectively
M_{ij*} moments about pin joint *A*, *B*, *C*, *D* and *O* at different acceleration *0*, *10*, *20*, *30* and *40 m/s*^{2} in the positive and negative crank rotation directions, where $i = a, b, c, d \quad and \quad o, j =$ *0*, *10*, *20*, *30* and *40 m/s*^{2}, and *= positive or negative crank rotation directions.
r_1, r_2, r_3, r_4 length of linkages *AB*, *BC*, *CD*, and *DO*, respectively
r_5 length of the vertical fixed link
r_6 length of horizontal fixed link
$x, \dot{x}, \ddot{x}$ linear displacement, linear velocity and linear acceleration respectively of the mechanism in *x* direction
$\beta_1, \dot{\beta}_1, \ddot{\beta}_1$ linkage *AB* angle, angular velocity and angular acceleration, respectively
$\beta_2, \dot{\beta}_2, \ddot{\beta}_2$ linkage *BC* angle, angular velocity and angular acceleration, respectively
$\beta_3, \dot{\beta}_3, \ddot{\beta}_3$ linkage *CD* angle, angular velocity and angular acceleration, respectively
$\beta_4, \dot{\beta}_4, \ddot{\beta}_4$ linkage *DO* angle, angular velocity and angular acceleration, respectively

1. Introduction

Some mechanical and biological systems do comprise of linkage mechanisms for the purpose of transfer of forces and motions from one point to another in a desired manner. The literature is saturated with synthesis, analysis, and findings in this area. Denavit and Hertenberg (1960), Mclarnan (1963), Rao (1981), and Freudenstein (1995) all worked on synthesis of linkages. Muller (1996) classified different types of linkages found in animals. Chavdarov (2005) used four spaces Jacoby Matrix for the analysis of kinematics and force of a five-bar link mechanism. Chang (2007) used modified Gear Method, a numerical method, to solve the dynamic equations generated in four bar linkage with clearance. Hoang et al (2015) studied the workings of a parallel robot based on 5-bar linkage. Essomba and Vu (2018) presented a new Spherical Decoupled Mechanism.

The models are simpler than majority of this type of decoupled mechanisms reported earlier. Their analysis demonstrated a very simple control and provided the suppression of the parallel singularity inside its workspace that the original five-bar spherical linkage suffers from and its operational workspace is consequently larger. This model also improved the kinematic performance for similar 5 bar linkage configurations. Gamble (2020) discussed and analyzed singularities, working modes, and error sensitivity and proposed a simple transform to approximate the forward and inverse kinematics in a 5-bar linkages. Sreenivasulu, et al (2021) described the design of a manipulator which operated by five bar parallel linkage mechanism and considered geometric approach to solve inverse kinematics developed using PYTHON codes.

Waghmare et. al. (2022) presented the performance of a teaching-learning-based optimization algorithm and its elite version named as an elitist teaching-learning-based optimization algorithm to obtain the optimum set of design parameters for the path synthesis of a four-bar linkage. The objective function is the minimization of the position error and they considered four case studies to verify the efficiency and accuracy of the algorithms. Translating mechanisms are mechanisms utilized on moving platforms but has not been examined. Applications include internal combustions engines, robotic arms, windshield wipers, forklift, a ride on a bicycle, 3D printers, laser cutters, CNC machines to mention but a few. The area of dynamics of translating five – bar linkage mechanisms has not been fully explored. It is therefore important to have a look at it. This will afford the designers, researchers and developers more insight into the principles and working of these mechanisms. It will also expose point of interest to note at design stage. The objective of this paper, therefore, is to present an analysis of a translating five – bar linkage mechanism.

2. Equation Derivations and Analysis

Figure 1 is the translating five-bar linkage mechanism with four active links and the fifth link fixed, a force is applied at point D that makes the crank linkage *DO* rotate clockwise. This mechanism is mounted on a moving platform and they both move together in the same positive x direction with velocity, $\dot{x}$and acceleration$\ddot{x}$. Since the velocity of the platform does not affect the rotation of the mechanism relative to the platform, the mechanism can be treated as a two degree of freedom system. Also, the overall positions of the linkages depend on the input angle of the crank linkage *DO*. The mechanism is assumed to be planar in planar motion. All joints are pins and are revolute.

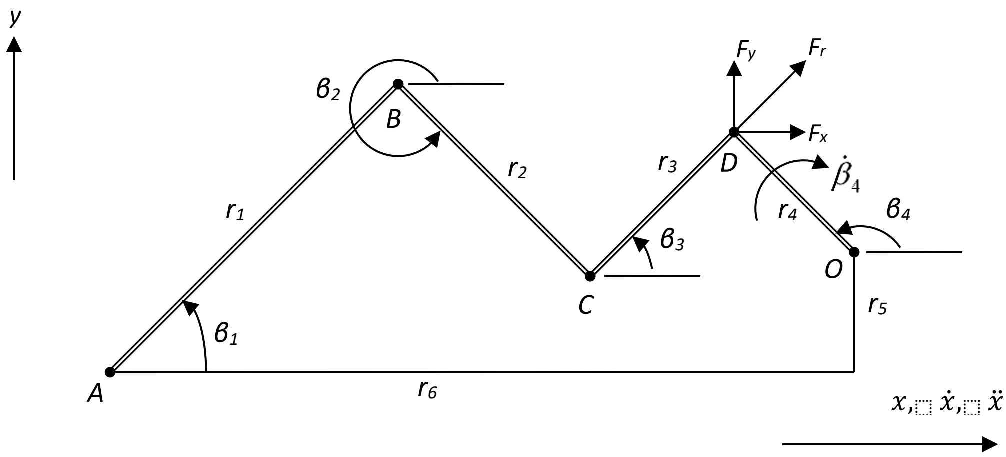

Figure 1 – Five – bar linkage mechanism

2.1 Position analysis

The position equations in the x and y directions can be written as equations 1a and 1b respectively:

$$r_1 \cos\beta_1 + r_2 \cos\beta_2 + r_3 \cos\beta_3 + r_4 \cos\beta_4 - r_6 = 0 \tag{1a}$$

$$r_1 \sin\beta_1 - r_2 \sin\beta_2 + r_3 \sin\beta_3 - r_4 \sin\beta_4 - r_5 = 0 \tag{1b}$$

2.2 Velocity analysis

Taking the first time-derivatives of Eq. (1a) and Eq. (1b) above and simplify, we obtain the velocity equations as expressed by equations 2a and 2b respectively:

$$r_1[-\sin\beta_1]\dot{\beta}_1 + r_2[-\sin\beta_2]\dot{\beta}_2 + r_3[-\sin\beta_3]\dot{\beta}_3 + r_4[-\sin\beta_4]\dot{\beta}_4 = 0 \tag{2a}$$

$$r_1[\cos\beta_1]\dot{\beta}_1 - r_2[\cos\beta_2]\dot{\beta}_2 + r_3[\cos\beta_3]\dot{\beta}_3 - r_4[\cos\beta_4]\dot{\beta}_4 = 0 \tag{2b}$$

The velocity equations of angles β_1 and β_2 are obtained from equations 2a and 2b above to give equations 3 and 4 thus

$$\dot{\beta}_1 = \frac{(r_4\dot{\beta}_4 \sin\beta_4 + r_3\dot{\beta}_3 \sin\beta_3)(-r_2\cos\beta_2) - (r_4\dot{\beta}_4\cos\beta_4 - r_3\dot{\beta}_3\cos\beta_3)(-r_2\sin\beta_2)}{-r_1\sin\beta_1(-r_2\cos\beta_2) - (r_1\cos\beta_1)(-r_2\sin\beta_2)} \quad (3)$$

$$\dot{\beta}_2 = \frac{(-r_4\dot{\beta}_4\cos\beta_4 - r_3\dot{\beta}_3\cos\beta_3)(-r_1\sin\beta_1) - (-r_4\dot{\beta}_4\sin\beta_4 + r_3\dot{\beta}_3\sin\beta_3)(r_1\cos\beta_1)}{-r_1\sin\beta_1(-r_2\cos\beta_2) - (r_1\cos\beta_1)(-r_2\sin\beta_2)} \quad (4)$$

2.3 Acceleration analysis

Taking the second time-derivates of position equations Eq. (1a) and Eq. (1b), to obtain the acceleration expression in equations 5(a) and 5(b) we have:

$$r_1(-\sin\beta_1)\ddot{\beta}_1 + r_2(-\sin\beta_2)\ddot{\beta}_2 = \begin{Bmatrix} r_2\dot{\beta}_2^2\cos\beta_2 + r_1(\dot{\beta}_1^2\cos\beta_1) + r_3\ddot{\beta}_3\sin\beta_3 \\ +r_3\dot{\beta}_3^2\cos\beta_3 - r_4\ddot{\beta}_4\sin\beta_4 - r_4\dot{\beta}_4^2\cos\beta_4 \end{Bmatrix} \quad (5a)$$

$$r_1(\cos\beta_1)\ddot{\beta}_2 - r_2(\cos\beta_2)\ddot{\beta}_2 = \begin{Bmatrix} r_1\dot{\beta}_1^2\sin\beta_1 - r_2\dot{\beta}_2^2\sin\beta_2 + r_3\dot{\beta}_3^2\sin\beta_3 \\ -r_3\ddot{\beta}_3\cos\beta_3 - l_4\beta_4^2\sin\beta_4 + l_4\ddot{\beta}_4\cos\beta_4 \end{Bmatrix} \quad (5b)$$

From the acceleration equations, Eq. (5a) and Eq. (5b), the expression for the acceleration of angles β_1 and β_2 are given in Eq. (6) and (7)

$$\ddot{\beta}_1 = \frac{\begin{Bmatrix} \begin{Bmatrix} r_2\dot{\beta}_2^2\cos\beta_2 + r_1(\dot{\beta}_1^2\cos\beta_1) + r_3\ddot{\beta}_3\sin\beta_3 \\ +r_3\dot{\beta}_3^2\cos\beta_3 - r_4\ddot{\beta}_4\sin\beta_4 - l_4\dot{\beta}_4^2\cos\beta_4 \end{Bmatrix} r_2(\cos\beta_2) \\ -\begin{Bmatrix} r_1\dot{\beta}_1^2\sin\beta_1 - r_2\dot{\beta}_2^2\sin\beta_2 + r_3\dot{\beta}_3^2\sin\beta_3 \\ -r_3\ddot{\beta}_3\cos\beta_3 - r_4\dot{\beta}_4^2\sin\beta_4 + l_4\ddot{\beta}_4\cos\beta_4 \end{Bmatrix} r_2(-\sin\beta_2) \end{Bmatrix}}{(-r_1\sin\beta_1)(-r_2\cos\beta_2) - r_1(\cos\beta_1)r_2(-\sin\beta_2)} \quad (6)$$

$$\ddot{\beta}_2 = \frac{\begin{Bmatrix} r_1(-\sin\beta_1)\begin{Bmatrix} r_1\dot{\beta}_1^2\sin\beta_1 - r_2\dot{\beta}_2^2\sin\beta_2 + r_3\beta_3^2\sin\beta_3 \\ -r_3\ddot{\beta}_3\cos\beta_3 - r_4\dot{\beta}_4^2\sin\beta_4 + r_4\ddot{\beta}_4\cos\beta_4 \end{Bmatrix} \\ -r_1(\cos\beta)\begin{Bmatrix} r_2\dot{\beta}_2^2\cos\beta_2 + r_1\dot{\beta}_1^2\cos\beta_1 + r_3\ddot{\beta}_3\sin\beta_3 \\ +r_3\dot{\beta}_3^2\cos\beta_3 - r_4\ddot{\beta}_4\sin\beta_4 - r_4\dot{\beta}_4^2\cos\beta_4 \end{Bmatrix} \end{Bmatrix}}{(-r_1\sin\beta_1)(-r_2\cos\beta_2) - r_1(\cos\beta_1)r_2(-\sin\beta_2)} \quad (7)$$

Figure 2 is the free body diagram of the five-bar linkage mechanism which is translating in the positive *x* direction. The motion of the platform will affect the reaction forces in the direction of the movement.

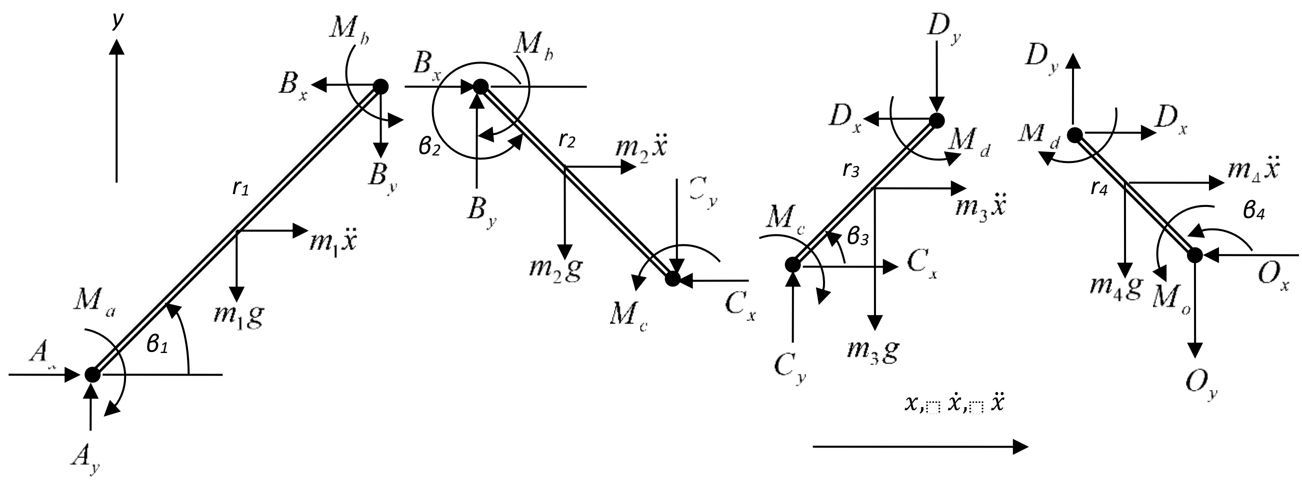

Figure 2 – Free body diagram of a five – bar linkage mechanism

2.4 Moments developed around the pin joints

The moment equations about the respective pin joints are determined by taking moments about the centre of gravity (CG) of the linkage and simplifying the equations thus:

Link *AB*: $M_a = -I_{ABCG}\ddot{\beta}_1 + M_b + a_1[(A_x + B_x)\, sin\,\beta_1 - (A_y + B_y)\, cos\,\beta_1]$ (8)

Link *BC*: $M_b = -I_{BCCG}\ddot{\beta}_2 + M_c + a_2[(B_x - C_x)\, cos\,\beta_2 - (B_y + C_y)\, sin\,\beta_2]$ (9)

Link *CD*: $M_c = -I_{CDCG}\ddot{\beta}_3 + M_d + a_3[(C_x + D_x)\, sin\,\beta_3 - (C_y + D_y)\, cos\,\beta_3]$ (10)

Link *DO*: $M_d = -I_{DOCG}\ddot{\beta}_4 + M_o + a_4[(D_x + O_x)\, cos\,\beta_4 - (D_y + O_y)\, sin\,\beta_4]$ (11)

2.5 Reaction forces at the pins

The joints are assumed to be revolute and frictionless. Therefore, there are no frictional forces developed at the joint. These forces are summed in the *x* and *y* positive directions and simplified to obtain as follows:

$\sum F_x = 0{:} \rightarrow +ve$ and $\sum F_y = 0{:} \uparrow +ve$ (12)

Linkage *CD*: $C_x = D_x - m_3\ddot{x}$ and $C_y = D_y + m_3 g$ (13)

Linkage *BC:*$B_x = C_x - m_2\ddot{x}$ and $B_y = C_y + m_2 g$ (14)

Linkage *AB:*$A_x = B_x - m_1\ddot{x}$ and $A_y = B_y + m_1 g$ (15)

Linkage *DO:*$O_x = D_x + m_4\ddot{x}$ and $O_y = D_y - m_4 g$ (16)

3. Observation and Discussions

The developed dynamic equations can be solved for any design purpose if the values of the parameters are known. The values of the parameters to be used for the simulations and discussions are given in Table 1.

Using the values of parameter given in Table 1 the horizontal reaction forces developed in Eq. (8 – 12) were determined and are plotted as shown in Figure 3 and Figure 4 for translating acceleration of *0, 10, 20, 30* and *40* m/s^2. It is observed in Figure 3 that the horizontal reaction forces A_x, B_x, and C_x decreases as the translating acceleration increases in the direction of crank rotation while O_x is increasing at a very small rate. In Figure 4, the horizontal reaction forces A_x, B_x, and C_x increases as the translating acceleration increases in the opposite direction of crank rotation while O_x is decreasing at a very small rate as well.

Table 1: Values parameters used for simulation (Adapted from: Redfield and Hull, 1996)

S/N	DESCRIPTION	SYMBOL	VALUES USED
1	Distance of CG from pin end of linkage *AB*	a_1	0.1715m
2	Distance of CG from pin end of linkage *BC*	a_2	0.2165m
3	Distance of CG from pin end of linkage *CD*	a_3	0.1015m
4	Distance of CG from pin end of linkage *DO*	a_4	0.085m
5	Length of linkage *AB*	r_1	0.343m
6	Length of linkage *BC*	r_2	0.433m
7	Length of linkage *CD*	r_3	0.203m
8	Length of linkage *DO*	r_4	0.170m
9	Length of vertical fixed link	r_5	0.212m
10	Length of horizontal fixed link	r_6	0.693m
11	Moments of inertia of linkage *AB* about CG	I_{ABCG}	6.013×10^{-2}
12	Moments of inertia of linkage *BC* about CG	I_{BCCG}	4.469×10^{-2}
13	Moments of inertia of linkage *CD* about CG	I_{CDCG}	5.56×10^{-3}
14	Mass of linkage *AB*	m_1	7.36kg
15	Mass of linkage *BC*	m_2	3.27kg
16	Mass of linkage *CD*	m_3	1.05kg
17	Mass of linkage *DO*	m_4	1.10kg
18	Acceleration due to gravity	g	$9.81 m/s^2$
19	Translating acceleration of the mechanism	$\ddot{x}$	$0,10,20,30,40 m/s^2$
20	Horizontal reaction force at pin D	D_x	$500N$
21	Vertical reaction force at pin D	D_y	$150N$

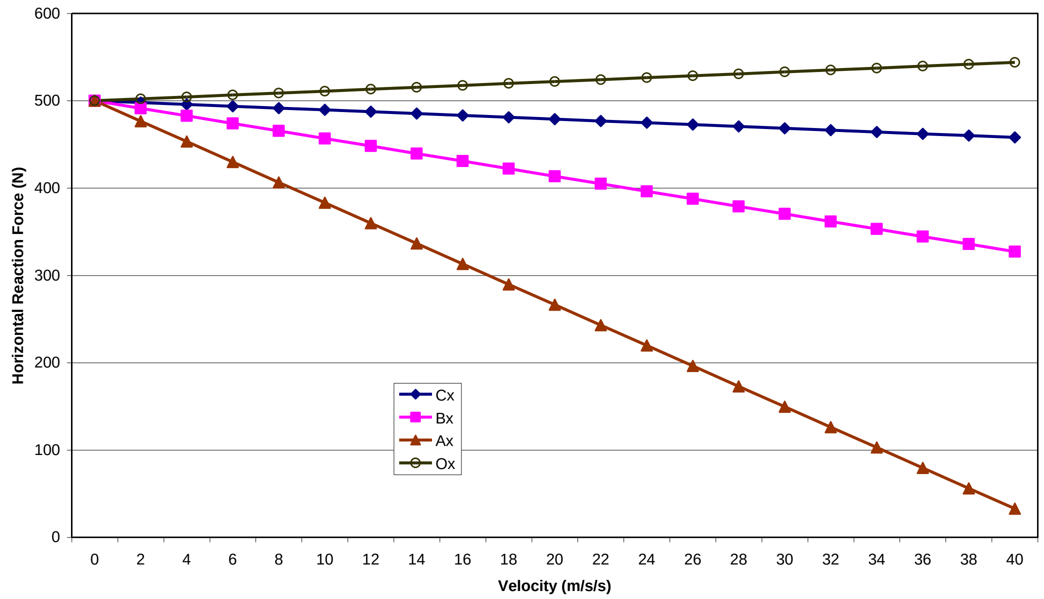

Figure 3 – Horizontal reaction forces and translating velocity in positive *x* direction

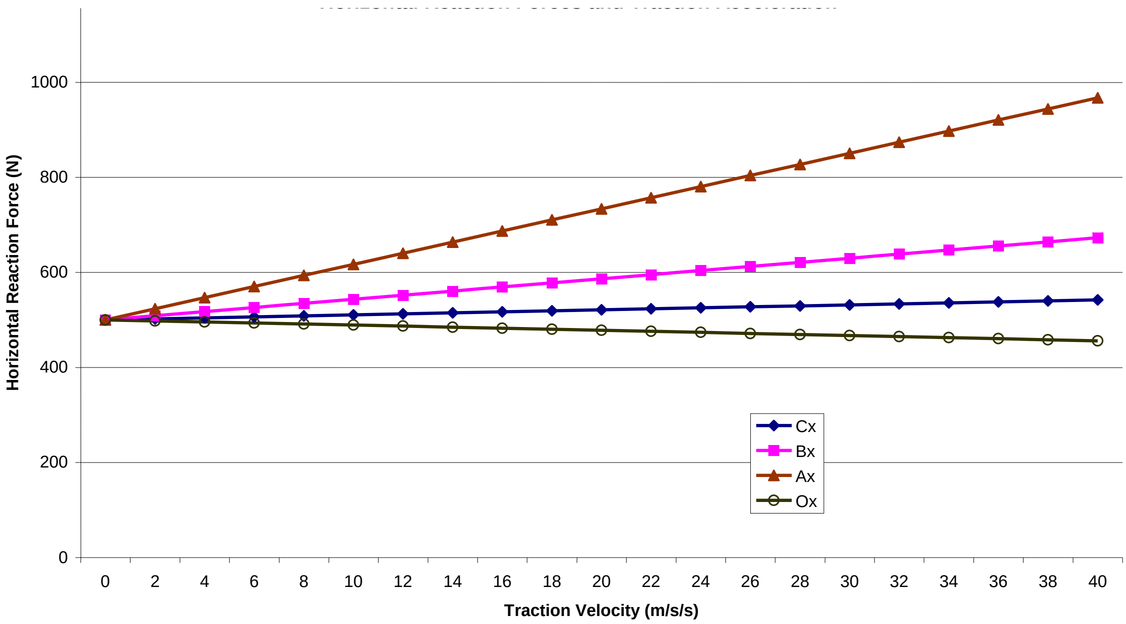

Figure 4 – Horizontal reaction forces and translating velocity in negative *x* direction

The parameter values in Table 1 were also used in Eq. (13 – 16) to obtain moments M_a, M_b, M_c and M_d at translating acceleration of *0*, *10*, *20*, *30* and *40 m/s²* both in the direction of crank rotation (positive x- direction) and the opposite (negative x-) direction as well. It was observed that, for pin joint moment M_a, Figure 5, decreases as the acceleration increases in the direction of crack rotation while it increases as the acceleration increases in the opposite direction to crank rotation direction. The pin joint moment M_b, shown in Figure 6 on the other hand increases as the acceleration increases in the direction of crank rotation but with lesser magnitude while it decreases in the opposite direction to the crank rotation. The pin joint moments M_c and M_d as shown in Figure 7 and Figure 8 does not show any significant variation in magnitude as the acceleration varied along the two directions.

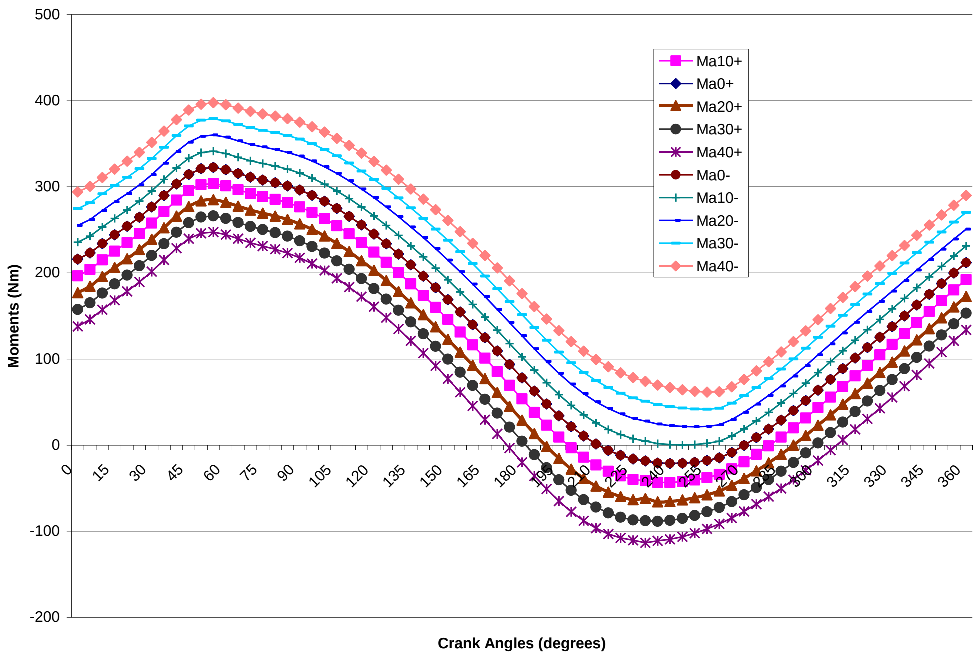

Figure 5 – Moments at pin A for translation velocities in the positive and negative *x* direction

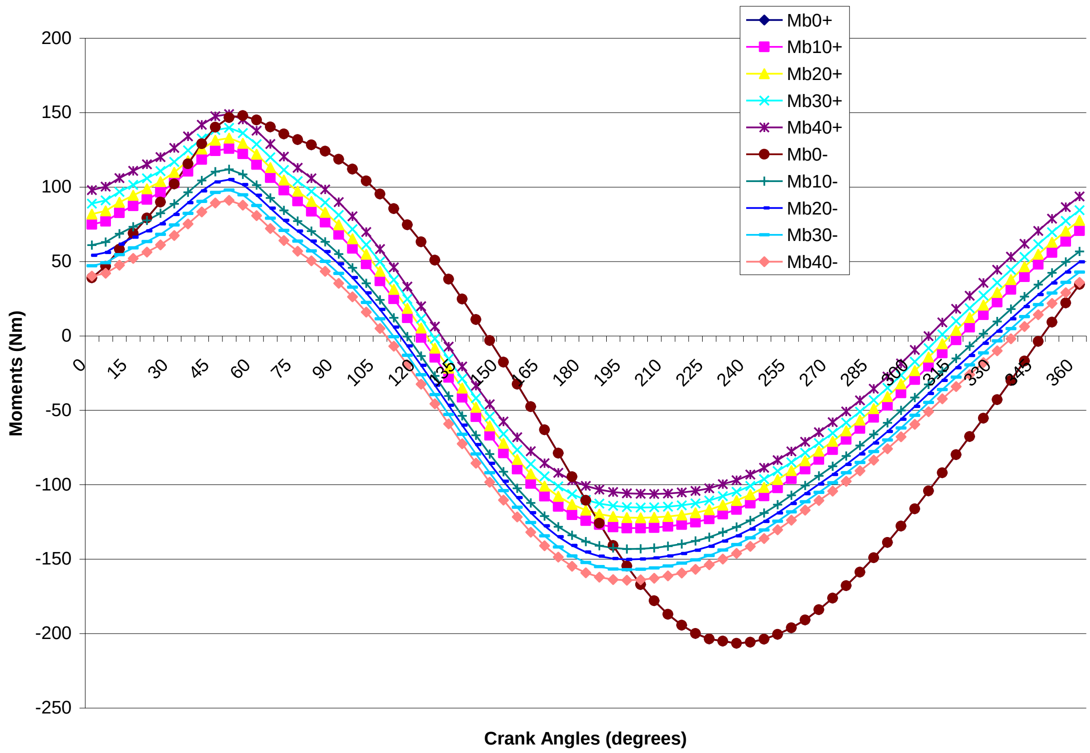

Figure 6 – Moments at pin *B* for translation velocities in the positive and negative *x* direction

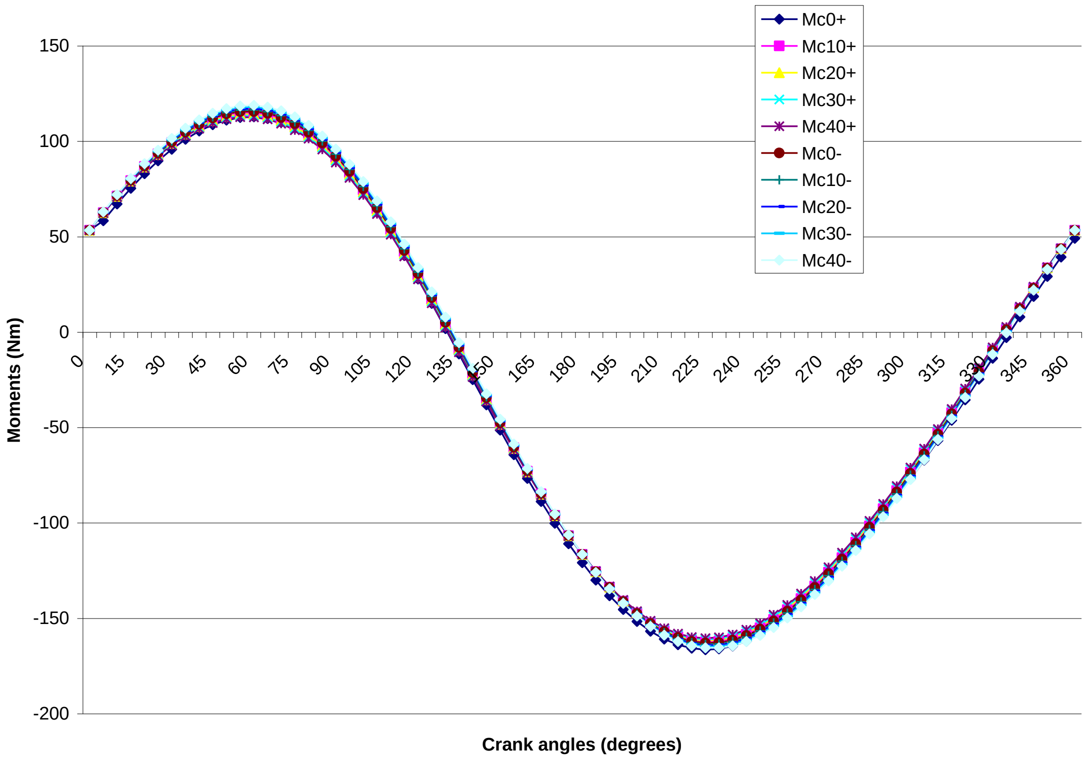

Figure 7 – Moments at pin *C* for translation velocities in the positive and negative *x* direction

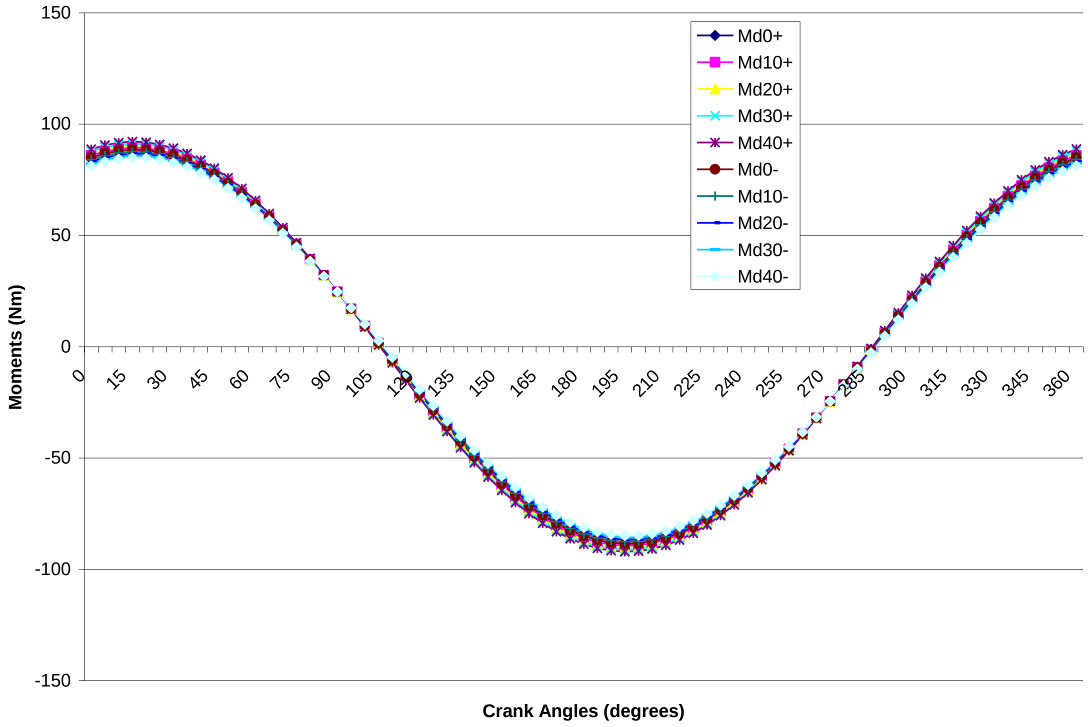

Figure 8 – Moments at pin *D* for translation velocities in the positive and negative *x* direction

4. Conclusions

The dynamic analysis of a five – bar linkage mechanism in translation has been presented. It was observed that the highest magnitude of reaction occurred at pin joint *A* when the mechanism is moving in the opposite direction of the crank arm rotation. The minimum in magnitude is obtained when it is moving in the direction of crank rotation, it eventually reaches zero at about $42.85 m/s^2$. The maximum pin joint moment, as expected, is developed at pin joint *A* when the acceleration is $40 m/s^2$ in the opposite direction to the crank rotation.

It can be concluded that in design and development of five – bar linkage mechanisms that will be utilized in a translation environment, it will be good if the crank rotation and the translation are so arranged to be in the same direction. This will reduce motion induced reactions and moments at the pin joints which can eventually lead to failures these points. The induced failure could manifest in the form of bearing overload and damage at this point. The mechanism can so be designed to have *zero* reaction at the pin joint *A* which will increase the integrity of the whole mechanism.

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