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Mass-Energy Equivalence derived from Newtonian mechanics

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Abstract:
In the theory of special relativity, the Einstein's relativistic mechanics is concerned with the motion of particles whose velocities approach the speed of light, but not equal to the speed of massless particle. It is understood the velocity of a moving particle with mass is less than the speed of velocity and velocity of a massless particle like photon is equal to the speed of light. This paper presents the Einstein's mass-energy equivalence and the equations of relativistic mass, momentum and energy from the Newton's second law of motion.

Keywords: relativistic motion, momentum equation, kinetic energy, rest mass.

1. Introduction

Newtonian mechanics is a mechanics that deals with concepts of Newton's law of motion including kinetic energy and potential energy. The equation of non-relativistic energy defined in the Newtonian mechanics states that $E = 0.5mv^2$, where m is non-relativistic mass and v is non-relativistic velocity. The equation of Einstein's mass-energy equivalence [Annamalai, 2023a, 2023b, 2023c, 2023d, 2023e, 2023f, 2023g, 2023h] is $E = mc^2$, where E, m, and c denote relativistic energy, mass, and speed of light, respectively.

2. Relativistic Mass, Momentum and Energy

The Einstein mass-energy equivalence denotes relativistic energy. In the theory of special relativity [Annamalai, 2023a, 2023g, 2023h] the relativistic momentum [Perez & Ribisi, 2022] is concerned with the motion of a particle whose velocity approaches the speed of light.

The Newton's second law of motion states that the force (F) acting on a particle is equal to the rate of change of its momentum (p). Let us derive the relativistic mass, momentum, and energy as follows.

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dv}{dt}, \text{where } v \text{ is velocity.} \quad (1)$$

The differential equation for work and kinetic energy is derived as follows:

$$dK = dW = Fds \tag{2}$$

$$dK = Fds = \left(m\frac{dv}{dt} + v\frac{dv}{dt}\right)ds \tag{3}$$

$$dK = Fds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dv, \text{where } \frac{ds}{dt} = v \tag{4}$$

Note that the term c^2dm allows the hypothesis of variable mass as it actually occurs at high speed. Also, c^2dm is equal to the kinetic energy.

$$dK = mvdv + v^2dm, \text{where } dK \text{ is kinetic energy.} \tag{5}$$

$$c^2dm = mvdv + v^2dm \tag{6}$$

$$\frac{dm}{m} = \frac{v}{c^2 - v^2}dv \tag{7}$$

$$\int_{m_0}^{m}\frac{dm}{m} = \int_0^v \frac{v}{c^2 - v^2}dv \tag{8}$$

$$[\ln(m)]_{m_0}^{m} = -\frac{1}{2}[\ln(c^2 - v^2)]_0^v \tag{9}$$

$$\ln m - \ln m_0 = -\frac{1}{2}\ln(c^2 - v^2) + \frac{1}{2}\ln c^2 \tag{10}$$

$$\ln\frac{m}{m_0} = \frac{1}{2}\ln\frac{c^2}{c^2 - v^2} \tag{11}$$

$$\frac{m}{m_0} = \sqrt{\frac{c^2}{c^2 - v^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{12}$$

$$\text{Relativistic mass } (m) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{13}$$

Where,

- the rest mass of the body is m_0,
- the velocity of the body in motion is v,
- the speed of the light is c.

$$\text{Relativistic momentum } (p) = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{Relativistic Energy } (E) = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

The relation between relativistic energy and momentum shows below:

$$E^2 = \frac{m_0^2 c^4}{1-\frac{v^2}{c^2}} \Longrightarrow E^2 = \frac{m_0^2 c^2 (v^2 - v^2 + c^2)}{1-\frac{v^2}{c^2}} \Longrightarrow E^2 = \frac{m_0^2 c^2 v^2 - m_0^2 c^2 v^2 + m_0^2 c^4}{1-\frac{v^2}{c^2}} \tag{14}$$

$$Then, E^2 = \left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)^2 c^2 + \frac{m_0^2 c^2 (c^2 - v^2)}{\frac{c^2 - v^2}{c^2}} \tag{15}$$

From the above expression, we obtain the energy-momentum relation

$$E^2 = p^2 c^2 + m_0^2 c^4. \tag{16}$$

If particle is at rest, then $p = 0$. Thus, the rest energy is that $E = m_0 c^2$.

Now, it is understood that the relativistic mass, momentum, and energy are derived from the Newton’s second law of motion.

3. Mass-Energy Equivalence
4.

$$\text{Relativistic mass } (m) = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \tag{17}$$

$$m^2 = \frac{{m_0}^2}{1-\frac{v^2}{c^2}} \Longrightarrow m^2(c^2 - v^2) = {m_0}^2 c^2, \text{where rest mass energy} = {m_0}^2 c^2$$

$$m^2 c^2 - m^2 v^2 = {m_0}^2 c^2 \tag{18}$$

By differentiating the equation with respect to time, we get

$$2mc^2 \frac{dm}{dt} - 2mv \frac{d(mv)}{dt} = 0 \Longrightarrow c^2 \frac{dm}{dt} = v \frac{d(mv)}{dt} \tag{19}$$

$$\frac{dE}{dt} = Fv = v \frac{d(mv)}{dt} = c^2 \frac{dm}{dt} \tag{20}$$

$$dE = c^2 dm \tag{21}$$

$$\int_0^K dK = \int_{m_0}^{m} c^2 dm \tag{22}$$

$K = c^2(m - m_0)$, where K is kinetic energy.

Total Energy (E) = Kinetic Energy (K) + Rest Mass-Energy ($m_0 c^2$)

$$E = c^2(m - m_0) + m_0 c^2 \tag{23}$$

$$E = c^2 m - c^2 m_0 + m_0 c^2 \tag{24}$$

$$E = c^2 m \tag{25}$$

$$\text{Therefore}, E = mc^2. \tag{26}$$

Hence, the equation of Einstein's mass-energy equivalence is derived from Newton's Second Law of Motion.

5. Conclusion

The mass-energy equivalence along with relativistic mass, momentum, and energy play an important role in the Einstein's theory of special relativity. Also, the relativistic mechanics is concerned with the motion of bodies whose velocities approach the speed of light. In this article, the equation of Einstein's mass-energy equivalence and the relativistic mass and momentum equation have been derived from Newton's Second Law of Motion.

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