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Explicit Exact Solutions of Nonlinear Transient Thermal Models of a Porous Moving Fin using Laplace transform - Exp-function method

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Abstract

The present paper develops explicit and non-power series exact solutions to the nonlinear heat transfer models of conductive-radiative-convective moving-porous fin using Laplace transform - Exp-function method, is presented. The developed solutions are employed for investigation of the included parameters on the transient and steady states studies of the moving-porous fin. The results submitted that the fin temperature is augmented with time increase due to increase heat transfer rate as time progresses. However, thermal parameter of the fin reduces from the its base to its tip. As the porosity, moving, convective-conductive-radiative parameters are increased, the fin temperature are decreased due to increased heat transfer rate. The opposite trend is displayed for the conductive-radiative number. It can be stated that present work will be useful in the analysis of the device.

Keywords: Porous fin. Thermal study. Explicit analytical solution. Laplace transform. Exp-function method.

1. Introduction

Fins have been extensively used for heat transfer augmentation in thermal systems [1]. Consequently, there have been different studies on their thermal responses when they are subjected to heat transfer [2-19]. The mathematical models of the thermal response of the passive devices are nonlinear which are very hard to be solved explicitly. Consequently, numerical methods and approximate analytical methods have been used [20-40]. In some earlier studies [21]-[27] some closed-formed solutions for the linearized thermal problem in the passive device were established.

Other authors [28]-[47] made used of other computational techniques to study the thermal characteristics of fins.

A critical look at the above review works show that the reviewed studies has not developed explicit analytical solutions for the passive device. However, as a novel contribution, we developed explicit and non-power series exact solutions to the nonlinear heat transfer models of conductive-radiative-convective moving porous fin using Laplace transform-exp(-Φ(x)-expansion method, is presented. The developed solutions are employed for investigation of the included parameters on the transient and steady states studies of the fin.

2. Problem Formulation

A moving straight moving radiative-convective porous fin influence by magnetic field as shown (Fig.1.) is considered. The assumption made are given in our previous studies [35-37].

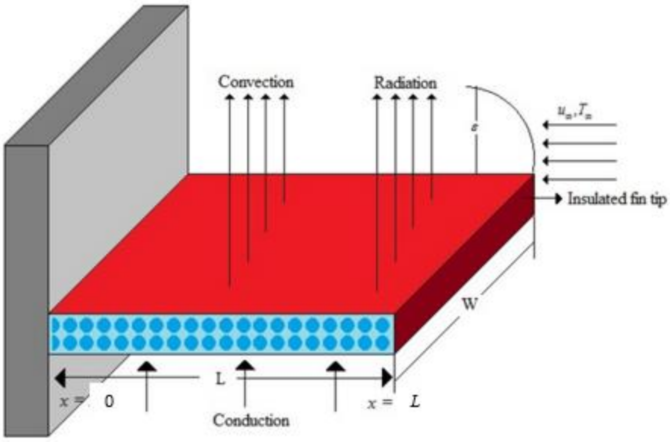

Fig. 1 The porous fin under convection and radiation

Using the energy balance, thermal transient model of the device is

$$\frac{\partial}{\partial \hat{x}^*}\left[k_{eff}\frac{\partial \hat{T}^*}{\partial \hat{x}^*}\right]+\frac{4\sigma}{3\beta_R}\frac{\partial}{\partial \hat{x}^*}\left(\frac{\partial \hat{T}^{*4}}{\partial \hat{x}^*}\right)-\frac{\beta^{'}\rho c_p gKP(\hat{T}^*-T_a^*)^2}{A_{cr}\nu_f}-\frac{Ph(\hat{T}^*-T_a^*)}{A_{cr}} -\frac{P\sigma\varepsilon(\hat{T}^{*4}-T_a^{*4})}{A_{cr}}=\rho c_p\frac{\partial \hat{T}^*}{\partial \hat{t}^*}+u\frac{\partial \hat{T}^*}{\partial \hat{x}^*} \tag{1}$$

The initial and boundary conditions

$$\hat{T}^*\left(\hat{x}^*,0\right)=T_a^* \quad \text{for} \quad 0<\hat{x}^*<L \tag{2a}$$

$$\frac{\partial \hat{T}^{**}\left(0,\hat{t}^*\right)}{\partial \hat{x}^*}=0 \tag{2b}$$

$$\hat{T}^*\left(L,\hat{t}^*\right)=T_b \tag{2c}$$

The definition of each variable used in the model is given in our previous works [35-37.59].

Assuming constant effective thermal conductivity:

$$\begin{aligned}&\frac{\partial^2\hat{T}^*}{\partial\hat{x}^{2*}}+\frac{4\sigma}{3\beta_R k_{eff}}\frac{\partial}{\partial\hat{x}^*}\left(\frac{\partial\hat{T}^{*4}}{\partial\hat{x}^*}\right)-\frac{\beta^{'}\rho c_p gKP(\hat{T}^*-T_a^*)^2}{A_{cr}k_{eff}\nu_f}-\frac{Ph(\hat{T}^*-T_a^*)}{A_{cr}k_{eff}}\\&-\frac{P\sigma\varepsilon(\hat{T}^{*4}-T_a^{*4})}{A_{cr}k_{eff}}=\frac{\rho c_p}{k_{eff}}\frac{\partial\hat{T}^*}{\partial\hat{t}^*}+\frac{u}{k_{eff}}\frac{\partial\hat{T}^*}{\partial\hat{x}^*}\end{aligned} \tag{3}$$

For the case considered, we take

$$\hat{T}^{*4}=T_a^{*4}+4T_a^{*3}\left(\hat{T}^*-T_a\right)+6T_a^{*2}\left(\hat{T}^*-T_a\right)^2+\ldots\cong 4T_a^{*3}\hat{T}^*-3T_a^{*4} \tag{4}$$

Put Eq. (4) into Eq. (3), gives

$$\begin{aligned}&\frac{\partial^2\hat{T}^*}{\partial\hat{x}^{2*}}+\frac{4\sigma}{3\beta_R k_{eff}}\frac{\partial}{\partial\hat{x}^*}\left(\frac{\partial\hat{T}^{*4}}{\partial\hat{x}^*}\right)-\frac{\beta^{'}\rho c_p gKP(\hat{T}^*-T_a^*)^2}{A_{cr}k_{eff}\nu_f}-\frac{Ph(\hat{T}^*-T_a^*)}{A_{cr}k_{eff}}\\&-\frac{4P\sigma\varepsilon T_a^{*3}(\hat{T}^*-T_a^*)}{A_{cr}k_{eff}}=\frac{\rho c_p}{k_{eff}}\frac{\partial\hat{T}^*}{\partial\hat{t}^*}+\frac{u}{k_{eff}}\frac{\partial\hat{T}^*}{\partial\hat{x}^*}\end{aligned} \tag{5}$$

Adopting dimensionless parameters in Eq. (6) to Eq. (5),

$$\begin{aligned}&X=\frac{\hat{x}^*}{L},\ \Theta=\frac{\hat{T}^*-T_a^*}{T_b-T_a^*},\tau=\frac{t}{t_{\max}}\ Rd=\frac{4\sigma_{st}T_a^{*3}}{3\beta_R k_{eff}},Nr^*=\frac{4P\sigma_{st}\varepsilon L^2T_a^{*3}}{A_{cr}k_{eff}},M^{*2}=\frac{PhL^2}{A_{cr}k_{eff}},\\&S_h^*=\frac{\rho c_p gK\beta(T_b-T_a^*)L^2}{k_{eff}\delta\nu_f},\quad \zeta^*=\frac{\rho c_p L^2}{k_{eff}\tau_{\max}},\quad Pe^*=\frac{\rho c_p uL}{k_{eff}}\end{aligned} \tag{6}$$

The dimensionless form of the model in Eq. (7) is

$$\left(1+4Rd\right)\frac{\partial^2\Theta^*}{\partial X^2}-S_h^*\Theta^{*2}-M^{*2}\Theta^*-Nr^*\Theta^*=\zeta^*\frac{\partial\Theta^*}{\partial\tau}+Pe^*\frac{\partial\Theta^*}{\partial X} \tag{7}$$

Alternatively expressed as

$$\frac{\partial^2\Theta^*}{\partial X^2}-S_h\Theta^{*2}-M^2\Theta^*-Nr\Theta^*=\zeta\frac{\partial\Theta^*}{\partial\tau}+Pe\frac{\partial\Theta^*}{\partial X} \tag{8}$$

where

$$\zeta=\frac{\zeta^*}{\left(1+4Rd\right)},\ M^2=\frac{M^{*2}}{\left(1+4Rd\right)},\ S_h=\frac{S_h^*}{\left(1+4Rd\right)},\ Nr=\frac{Nr^*}{\left(1+4Rd\right)},\ Pe=\frac{Pe^*}{\left(1+4Rd\right)},$$

The dimensionless initial condition

$$\Theta^*\left(X,0\right)=0\quad\text{for}\quad 0<x^*<L \tag{9}$$

The dimensionless boundary conditions

$$\frac{\partial \Theta^*(0,\tau)}{\partial X} = 0 \tag{10a}$$

$$\Theta^*(1,\tau) = 1 \tag{10b}$$

3. Method of Solution: Laplace transform-Exp(-Φ(x)-Expansion Method

Laplace transform-exp(-Φ(x)-expansion method is utilized to develop explicit non-power series exact solutions to the models. The description of the Laplace has been given in our previous work [59]. However, we describe the exp(-Φ(x)-expansion method here.

3.1. Description of the Exp(-Φ(x)-Expansion Method

Exp-function method is a recently proposed method by He and Wu [48] is an effective, concise and straight-forward method for developing generalized solitary solutions and periodic solutions. This method has been widely applied in solving nonlinear problems [48-58]. The method gives more general solutions with some free parameters which make the development of exact solution relatively easy.

Consider an ordinary differential equation as

$$R(u,u',u'',u''',...) = 0 \tag{11}$$

where $u',u'',u''',...$ denote derivatives of u with respect to X and R is a polynomial of u. On integrating the ODE in Eq. (11) as many times as possible and set the constants of integration to zero.

The solution of ODE in Eq. (2) can be expressed by a polynomial in *exp(-Φ(X))* as

$$\Theta(X) = \sum_{i=0}^{N} A_i \left[exp\left(-\Phi(X)\right)\right]^i \tag{12}$$

Where A_i are constants to be determined such that $A_N \neq 0$ and Φ=Φ(x) satisfies the following ODE :

$$\Phi'(X) = exp\left(-\Phi(X)\right) + \mu exp\left(\Phi(X)\right) + \lambda \tag{13}$$

The above Eq.(14) gives the following solutions:

(i) When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$\Phi(X) = In\left[\frac{-\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\dfrac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(x+E)\right]-\lambda}{2\mu}\right] \tag{14}$$

(ii) When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$\Phi(X) = In\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}tanh\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(x+E)\right]-\lambda}{2\mu}\right] \tag{15}$$

(iii) When $\mu = 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu > 0,$

$$\Phi(X) = -In\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right] \tag{16}$$

(iv) When $\mu \neq 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(X) = In\left[-\frac{2\left[\lambda\left(\lambda+E\right)\right]+2}{\lambda^2(X+E)}\right] \tag{17}$$

(v) When $\mu = 0,\ \lambda = 0$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(X) = In(X+E) \tag{18}$$

Where $A_N,...,E,\lambda,\mu$ are constants to be determined latter, $A_N \neq 0$.

Substituting Eq.(11) into Eq. (12) gives a polynomial of $exp\left(-\Phi(X)\right)$ and equating all coefficients of the same power of $exp\left(-\Phi(X)\right)$ to zero gives a system of algebraic equations whichever can be solved to find $A_N,...,E,\lambda,\mu$. Also, substituting the value of $A_N,...,E,\lambda,\mu$ into Eq. (12) along the general solutions of Eq. (4) completes the determination of the solution of Eq. (11).

4. Development of Explicit analytical solutions for the Thermal Model using Laplace transform-Exp(-Φ(x)-expansion method

In order to find exact solution for the nonlinear model as derived in Eq. (15), we first apply Laplace transform to Eq. (8), which gives

$$\frac{d^2\tilde{\Theta}}{dX^2} - Pe\frac{d\tilde{\Theta}}{dX} - S_h\tilde{\Theta}^2 - \left(\zeta s + M^2 + Nr\right)\tilde{\Theta} = 0 \tag{19}$$

And the boundary conditions in Laplace domain are

$$s > 0, \quad X = 0, \quad \tilde{\Theta} = \frac{1}{s} \tag{20a}$$

$$s>0,\quad X=1,\quad \frac{\partial\tilde{\Theta}}{\partial x}=0 \tag{20b}$$

Now, for the governing equation gives in Eq. (11), we get N=2 (second-order ODE) if we follow homogenous balancing phenomena, therefore the suggested algorithms have the following solutions:

$$\tilde{\Theta}(X,s)=A_0+A_1 exp\left(-\bar{\Phi}(X,s)\right)+A_2\left[exp\left(-\bar{\Phi}(X,s)\right)\right]^2 \tag{21}$$

Where A_0, A_1 are constants to be determined such that $A_N \neq 0$, while λ, μ are arbitrary constants.

Substituting Eq. (21) into Eq. (8) and equating the coefficients of $exp\left(-\bar{\Phi}(X,s)\right)^4$, $exp\left(-\bar{\Phi}(X,s)\right)^3$, $exp\left(-\bar{\Phi}(X,s)\right)^2$, $exp\left(-\bar{\Phi}(X,s)\right)$, $exp\left(-\bar{\Phi}(X,s)\right)^0$ to zero, we respectively obtain.

$$\begin{aligned}
&exp\left(-\bar{\Phi}(X,s)\right)^4: \quad -S_hA_2^2+6A_2=0\\
&exp\left(-\bar{\Phi}(X,s)\right)^3: \quad 2PeA_2-2A_1A_2+10A_2\lambda+2A_1=0\\
&exp\left(-\bar{\Phi}(X,s)\right)^2: \quad 2PeA_1-2S_hA_0A_2+2PeA_2\lambda+4A_2\lambda^2+8A_2\mu-\left(\zeta s+M^2+Nr\right)A_2-S_hA_1^2+3A_1\lambda=0\\
&exp\left(-\bar{\Phi}(X,s)\right)^1: \quad -\left(\zeta s+M^2+Nr\right)A_1+6A_2\lambda\mu-2S_hA_0A_1+A_1\lambda^2+PeA_1\lambda+2PeA_2\mu+2A_1\mu=0
\end{aligned}$$

$$exp\left(-\bar{\Phi}(X,s)\right)^0: \quad -\left(\zeta s+M^2+Nr\right)A_0+PeA_1\mu-S_hA_0^2+A_1\mu\lambda+2A_2\mu^2=0$$

On solving the algebraic equation, we have

Cluster 1:

$$\begin{aligned}
&\mu=\frac{-A_1S_h}{144}\left(10\left(\zeta s+M^2+Nr\right)R_3-S_hA_1\right),\ \lambda=-\frac{1}{30R_3}\left(6-5S_hA_1R_3\right),\\
&A_0=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right),\\
&A_1=A_1,\ A_1=\frac{6}{S_h},\ R_3=\pm\sqrt{\frac{6}{25\left(\zeta s+M^2+Nr\right)}}=\frac{1}{Pe}
\end{aligned} \tag{22}$$

Cluster 2:

$$\begin{aligned}
&\mu=\frac{A_1S_h}{144}\left(10\left(\zeta s+M^2+Nr\right)iR_3+S_hA_1\right),\quad \lambda=\frac{1}{30iR_3}\left(6-5S_hA_1iR_3\right),\quad A_0=\frac{1}{24}S_hA_1^2,\\
&A_1=A_1,\ A_1=\frac{6}{S_h},\ R_3=\pm\sqrt{\frac{6}{25\left(\zeta s+M^2+Nr\right)}}=\frac{i}{Pe}
\end{aligned} \tag{23}$$

Substituting Eqs. (22) and (23) into Eq. (21), we have the solutions for the two clusters

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)+A_1 exp\left(-\bar{\Phi}(X,s)\right)+\frac{6}{S_h}\left[exp\left(-\bar{\Phi}(X,s)\right)\right]^2 \qquad (24)$$

And

$$\tilde{\Theta}(X,s)=\frac{1}{24}S_hA_1^2+A_1 exp\left(-\bar{\Phi}(X,s)\right)+\frac{6}{S_h}\left[exp\left(-\bar{\Phi}(X,s)\right)\right]^2 \qquad (25)$$

Substituting Eq. (14)-(18) into Eqs. (24), we have for **Cluster 1**

(i) When $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)-A_1\left[\frac{2\mu}{-\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]-\lambda}\right]$$
$$+\frac{6}{S_h}\left[\frac{2\mu}{-\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]-\lambda}\right]^2 \qquad (26)$$

(ii) When $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)+A_1\left[\frac{2\mu}{\sqrt{\left(4\mu-\lambda^2\right)}tanh\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(X+E)\right]+\lambda}\right]$$
$$+\frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(4\mu-\lambda^2\right)}tanh\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(X+E)\right]+\lambda}\right]^2 \qquad (27)$$

(iii) When $\mu=0,\ \lambda\neq 0$ and $\lambda^2-4\mu>0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)+A_1\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]+\frac{6}{S_h}\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]^2 \qquad (28)$$

(iv) When $\mu\neq 0,\ \lambda\neq 0$ and $\lambda^2-4\mu=0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)-A_1\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]+\frac{6}{S_h}\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]^2 \qquad (29)$$

(v) When $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(\zeta s+M^2+Nr\right)\right)+\frac{A_1}{(X+E)}+\frac{6}{S_h(X+E)^2} \tag{30}$$

Substituting Eq. (14)-(18) into Eqs. (25), we have for **Cluster 2**

(i) When $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$\begin{aligned}\tilde{\Theta}(X,s)=\frac{1}{24}S_h^2A_1^2-A_1&\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]\\&+\frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]^2\end{aligned} \tag{31}$$

(ii) When $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$\begin{aligned}\tilde{\Theta}(X,s)=\frac{1}{24}S_hA_1^2+A_1&\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]\\&+\frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]^2\end{aligned} \tag{32}$$

(iii) When $\mu=0,\ \lambda\neq 0$ and $\lambda^2-4\mu>0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24}S_hA_1^2+A_1\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]+\frac{6}{S_h}\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]^2 \tag{33}$$

(iv) When $\mu\neq 0,\ \lambda\neq 0$ and $\lambda^2-4\mu=0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}S_hA_1^2-A_1\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]+\frac{6}{S_h}\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]^2 \tag{34}$$

(v) When $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0,$

$$\tilde{\Theta}(X,s)=\frac{1}{24S_h}S_hA_1^2+\frac{A_1}{(X+E)}+\frac{6}{S_h(X+E)^2} \tag{35}$$

where E and A_1 are arbitrary constants to be determined from the boundary conditions.

$$\mu = \mu\left(s, S_h, M, Nr, Pe\right) \text{ and } \lambda = \lambda\left(s, S_h, M, Nr, Pe\right) \tag{36}$$

The numerical inverse Laplace transforms are carried out by applying the Simon's approach

$$\Theta(X,\tau) = \frac{e^{a_p\tau}}{\tau}\left[\frac{1}{2}\tilde{\Theta}(X,a_p) + \sum_{n=1}^{N} Re\left[\tilde{\Theta}(X,a_p + i\frac{n\pi}{\tau})\right](-1)^n\right] \tag{37}$$

The above solutions are for the transient heat transfer in the fins. However, as $\tau \to \infty$, a steady state is reached where the temperature distribution in the fin is invariant of time. Therefore, the solution of the steady state heat transfer in the fin are given as follows:

For the Cluster 1

(i) When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$\Theta(X) = \frac{1}{24S_h}\left(S_h^2A_1^2 - 24\left(M^2 + Nr\right)\right) - A_1\left[\frac{2\mu}{-\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]-\lambda}\right] + \frac{6}{S_h}\left[\frac{2\mu}{-\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]-\lambda}\right]^2 \tag{38}$$

(ii) When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$\Theta(X) = \frac{1}{24S_h}\left(S_h^2A_1^2 - 24\left(M^2 + Nr\right)\right) + A_1\left[\frac{2\mu}{\sqrt{\left(4\mu-\lambda^2\right)}tanh\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(X+E)\right]+\lambda}\right] + \frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(4\mu-\lambda^2\right)}tanh\left[\frac{\sqrt{\left(4\mu-\lambda^2\right)}}{2}(X+E)\right]+\lambda}\right]^2 \tag{39}$$

(iii) When $\mu = 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu > 0,$

$$\Theta(X) = \frac{1}{24S_h}\left(S_h^2A_1^2 - 24\left(M^2 + Nr\right)\right) + A_1\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right] + \frac{6}{S_h}\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]^2 \tag{40}$$

(iv) When $\mu \neq 0,\ \lambda \neq 0$ and $\lambda^2 - 4\mu = 0,$

$$\Theta(X)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(M^2+Nr\right)\right)-A_1\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]+\frac{6}{S_h}\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]^2 \tag{41}$$

(v) When $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0,$

$$\Theta(X)=\frac{1}{24S_h}\left(S_h^2A_1^2-24\left(M^2+Nr\right)\right)+\frac{A_1}{(X+E)}+\frac{6}{S_h(X+E)^2} \tag{42}$$

where E and A_1 are arbitrary constants to be determined from the boundary conditions.
$\mu=\mu\left(S_h,M,Pe\right)$ and $\lambda=\lambda\left(S_h,M,Pe\right)$

While for the **Cluster 2**

(i) When $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$\Theta(X)=\frac{1}{24}S_h^2A_1^2-A_1\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]+\frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]^2 \tag{43}$$

(ii) When $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$\Theta(X)=\frac{1}{24}S_hA_1^2+A_1\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]+\frac{6}{S_h}\left[\frac{2\mu}{\sqrt{\left(\lambda^2-4\mu\right)}tanh\left[\frac{\sqrt{\left(\lambda^2-4\mu\right)}}{2}(X+E)\right]+\lambda}\right]^2 \tag{44}$$

(iii) When $\mu=0,\ \lambda\neq 0$ and $\lambda^2-4\mu>0,$

$$\Theta(X)=\frac{1}{24}S_hA_1^2+A_1\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]+\frac{6}{S_h}\left[\frac{\lambda}{exp\left[\lambda(x+E)-1\right]}\right]^2 \tag{45}$$

(iv) When $\mu\neq 0,\ \lambda\neq 0$ and $\lambda^2-4\mu=0,$

$$\Theta(X)=\frac{1}{24S_h}S_hA_1^2-A_1\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]+\frac{6}{S_h}\left[\frac{\lambda^2(X+E)}{2\left[\lambda\left(\lambda+E\right)\right]+2}\right]^2 \qquad (46)$$

(v) When $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0$,

$$\Theta(X)=\frac{1}{24S_h}S_hA_1^2+\frac{A_1}{(X+E)}+\frac{6}{S_h(X+E)^2} \qquad (47)$$

where E and A_1 are arbitrary constants to be determined from the boundary conditions:

$\mu=\mu\left(S_h,M,Nr,Pe\right)$ and $\lambda=\lambda\left(S_h,M,Nr,Pe\right)$

4. Results and Discussion

Studies of the impacts of porosity, moving, convection-conduction-radiation parameters on the fins temperature are displayed in Figs. 2-8.

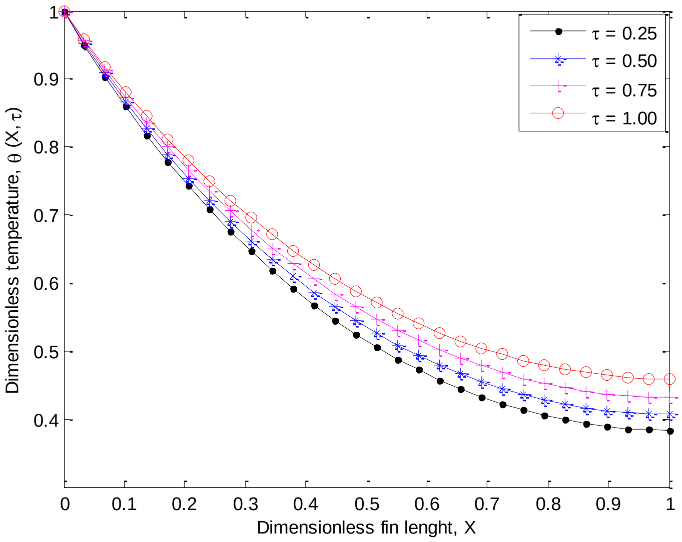

Fig. 2 Fin temperature at different time

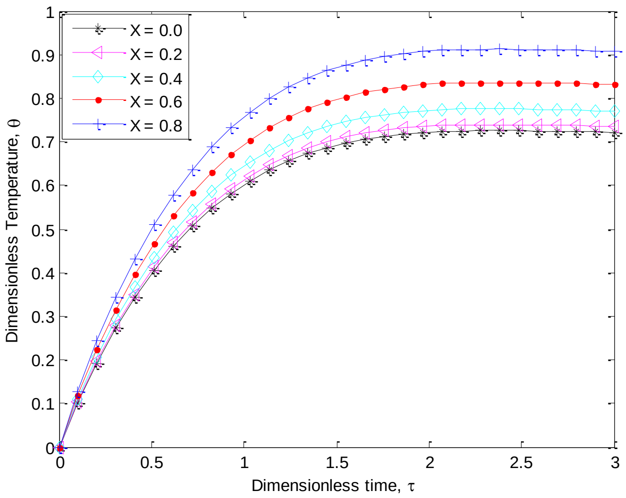

Fig. 3 Fin temperature histories at different points

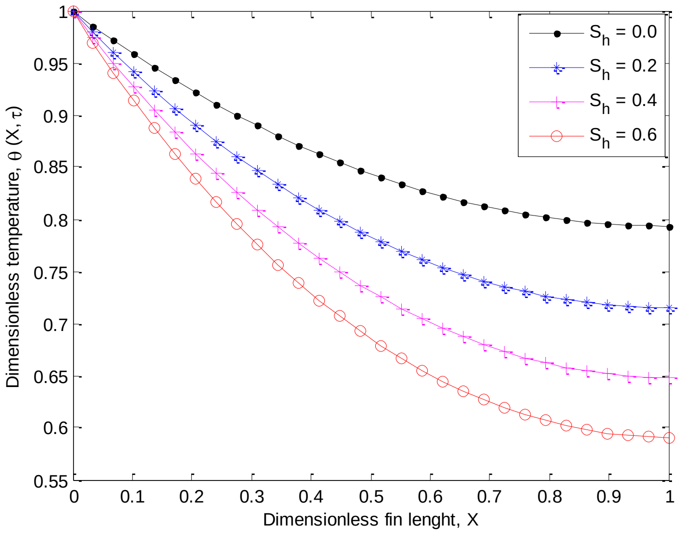

Fig. 4 Fin temperature for different porous parameters

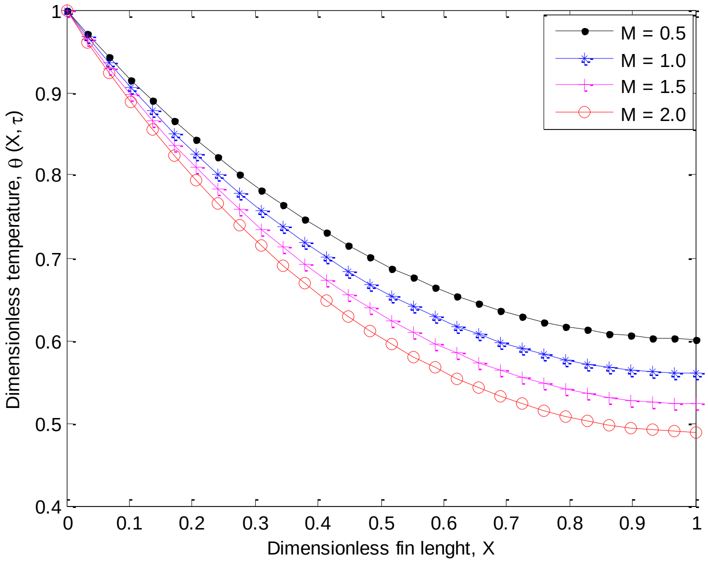

Fig. 5 Fin temperature for different convective parameters

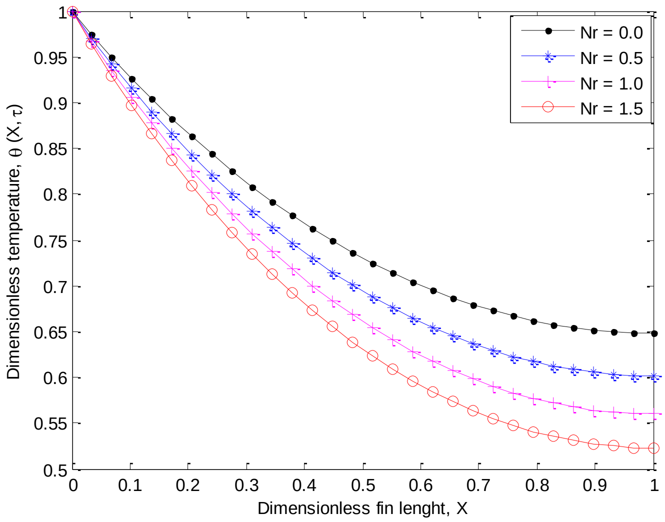

Fig. 6 Fin temperature for different radiative parameters

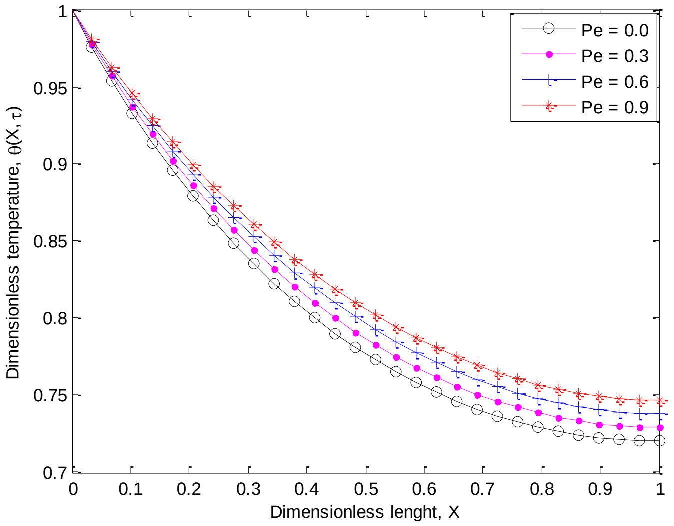

Fig. 7 Fin temperature for different moving parameters

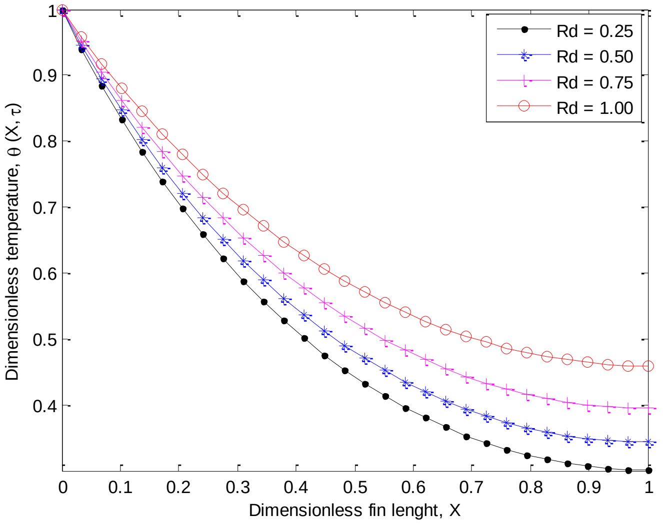

Fig. 8 Fin temperature for different conductive-radiative number

The thermal profiles of the device at various times and points are shown in Figs. 2 and 3. The fin temperature is augmented with time increase due to increase heat transfer rate as time progresses. However, thermal parameter of the fin reduces from the fin base to its tip as shown in Fig. 3. Figs. 4-7 showcase the influences porosity, moving, convective-conductive-radiative parameters on the thermal profiles of the extended surface. As display, these parameters have great significant roles on the heat transfer augmentations in the fin. The figures present that when the porosity, moving, convective-conductive-radiative parameters are increased, the fin temperature are decreased due to increased heat transfer rate. In Fig. 8, the opposite trend is displayed for the conductive-radiative number.

5. Conclusion

The work has developed explicit exact solution for transient and steady state heat transfer in moving-porous fins using Laplace transform-exp(-Φ(x)-expansion method. The developed solutions have been used investigate the impacts of the included parameters on the transient and steady states of the performance of the moving-porous fin. The results presented that the fin temperature is augmented with time increase. However, thermal parameter of the fin reduces from its base to its tip. When the porosity, moving, conducting-convective-radiative parameters are increased, the fin temperature are decreased due to increased heat transfer rate. The opposite trend is displayed for the conductive-radiative number. The work will be used for analysis and design of the device.

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